The New Mathematics
for Primary Teachers

6. Relations and Graphs

D. Paling and J. L. Fox

OXFORD UNIVERSITY PRESS

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FOR PRIMARY TEACHERS

/c

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Preface to the series

At the present time there is a growing dissatisfaction with mathematical work in schools. There is a feeling that the content of courses does not fit in with the present-day needs of science and technology, industry and trade. At the same time the attitude of teachers towards the teaching of mathematics is undergoing a fundamental change. More and more teachers are coming to feel that teaching by 'rule' is not good enough. They want children to find out more for themselves, to question what they are doing and to find their own ways of solving problems. They want children to understand rather than to learn like parrots.

These two movements—the modernisation of the content of mathematics and the shift of emphasis from 'the teacher teaching' to 'the child learning'—are going on together in many countries. They both make large demands on teachers, especially in the primary school. These teachers now need to take a new look at mathematics itself and at the same time try to introduce more enlightened methods in the classroom.

If these changes are to come about, teachers in schools and students in training will need all the help they can be given. It is for this purpose that this series of short books has been written. The aim is to give teachers and student-teachers a new mathematical background so that they can introduce modern mathematics in the primary school. The books do not set out to deal with the classroom situation itself but they do present, at teacher level, the kind of approach now being introduced into schools.

The completed set of books will provide teachers with a mathematical background for their work in the primary school. The approach is simple and straightforward and no previous knowledge is assumed. Many exercises (with answers) are provided at each stage; they need to be worked through very carefully. The exercises not only consolidate the work already done but provide the starting-point for the next stage.

St Luke's College, Exeter
October, 1971

D.P.
J.L.F.

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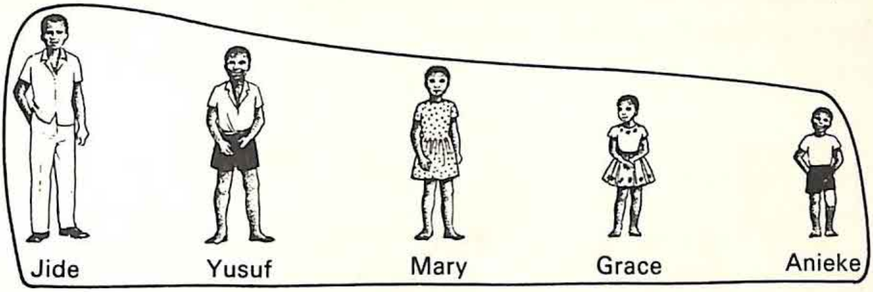
1. SHOWING RELATIONSHIPS

A. Relationships between the members of a set



Look at the set of children. They are arranged in order of height. We can make statements such as:

Jide *is taller than* Yusuf.
Jide *is taller than* Mary.
Jide *is taller than* Grace.
Jide *is taller than* Anieke.
Yusuf *is taller than* Mary.
Yusuf *is taller than* Grace.
Yusuf *is taller than* Anieke.
Mary *is taller than* Grace.
Mary *is taller than* Anieke.
Grace *is taller than* Anieke.

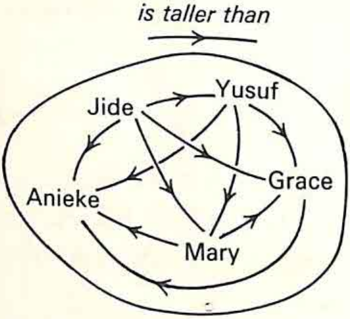
To write the full set of statements takes a long time. We can save some time by replacing *is taller than* in each statement by an arrowed line. The set of statements then becomes:

is taller than
→

Jide → Yusuf
Jide → Mary
Jide → Grace
Jide → Anieke
Yusuf → Mary
Yusuf → Grace
Yusuf → Anieke
Mary → Grace
Mary → Anieke
Grace → Anieke

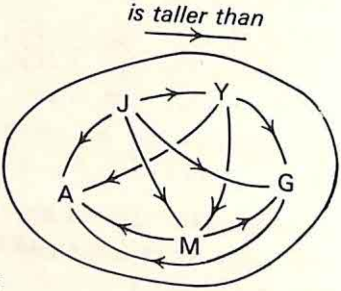
We show, above the set, what the arrowed line represents.

We can save more time and space by writing the set of names and then drawing the arrowed lines, like this:



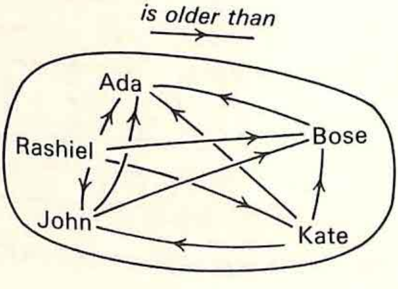
Again we show what the arrowed line represents. A diagram like that above is often called an **arrow graph**.

We can, of course, shorten our work even more by writing only the first letter of each name. The arrow graph then looks like this:



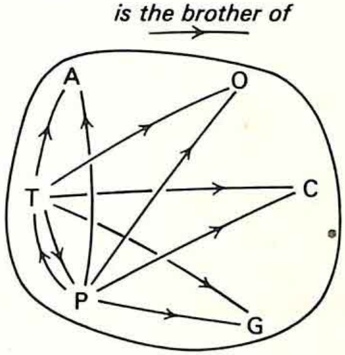
Exercises

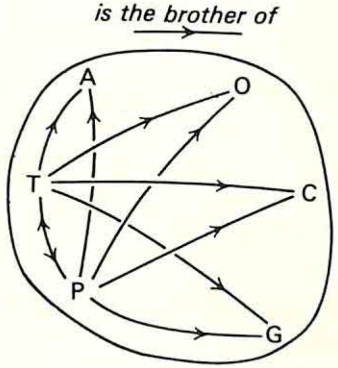
1. In the graph above:
   (a) from which child do most arrows go?
   (b) to which child do most arrows come?
   What do the answers tell you?
2. Draw an arrow graph for {Jide, Yusuf, Grace, Mary, and Anieke} to show *is shorter than*.
3. Look at the arrow graph. Which child is:
   (a) the oldest?
   (b) the youngest?



Mr and Mrs Akinleye have six children. There are four girls (Anna, Omono, Caro, and Grace) and two boys (Paul and Tani).

Here are two ways of showing, for the children, *is the brother of*.





In the first graph two arrowed lines are drawn between T and P. One of these shows that T *is the brother of* P. The other shows that P *is the brother of* T.

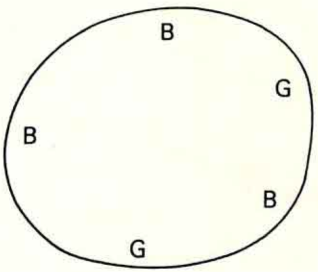
In the second graph the two statements are shown by putting two arrow heads on one line.

Both of these graphs are correct. We can draw either. In mathematics, however, the second kind of graph is used more often. So, for this reason, we shall use it.

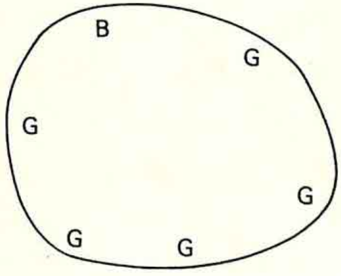
Exercises

4. Draw an arrow graph to show *is the brother of* for the family of children below (B shows a boy; G shows a girl):

(a)

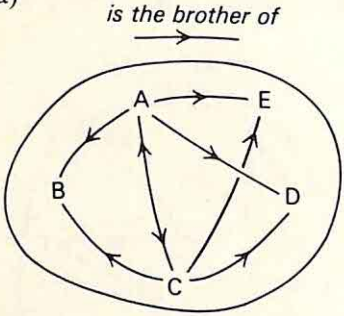


(b)

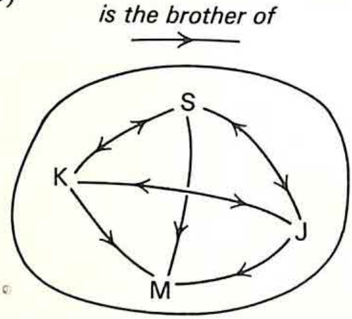


5. Here are some arrow graphs of other families for *is the brother of*. How many boys are there in each family?

(a)

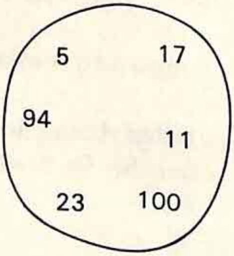


(b)

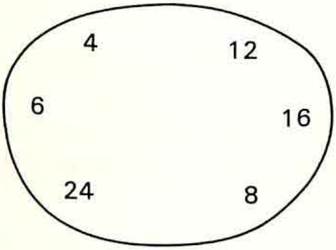


6. For each of the two families in exercise 5 draw a graph to show *is the sister of*.

7. Draw an arrow graph to show *is six more than* for this set of numbers.

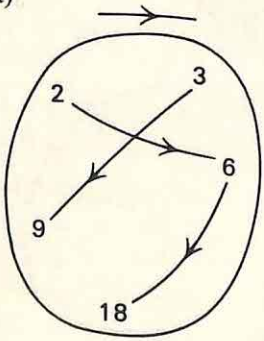


8. Draw an arrow graph to show *is a half of* for this set of numbers.

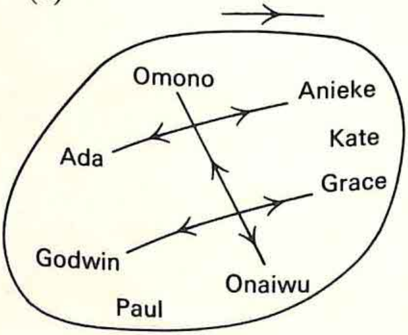


9. What can be written for the arrowed line?

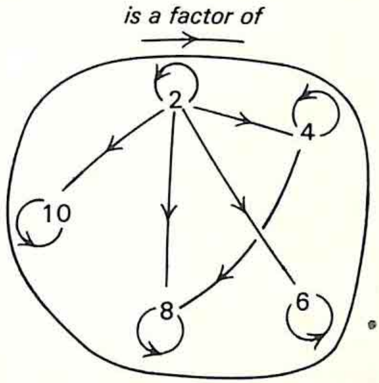
(a)



(b)



10. Here is a graph of {2, 4, 6, 8, 10} for *is a factor of*.

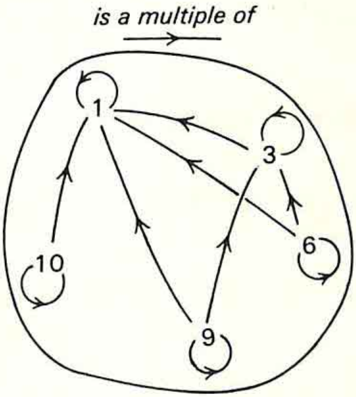


Note: If a number divides another number exactly, the first number is a factor of the second (e.g. 14 ÷ 2 = 7, so 2 is a factor of 14).

Why do you think a loop, ⟲, is drawn for each number?

Is it true that 2 *is a factor of* 2 and that 4 *is a factor of* 4?

11. Here is a graph of {1, 3, 6, 9, 10} for *is a multiple of*.



Note: If a number is multiplied by a second number, the result is a multiple of each of the two numbers (e.g. 2 × 7 = 14, so 14 is a multiple of 2 and a multiple of 7).

Why do you think that a loop is drawn for each number?

Is it true that 3 *is a multiple of* 3?

12. Draw an arrow graph for {2, 4, 6, 8, 12} to show *is a factor of*.

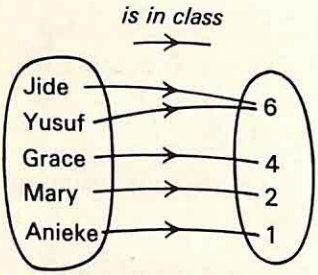
13. Draw an arrow graph for {1, 5, 10, 15, 20} to show *is a factor of*.

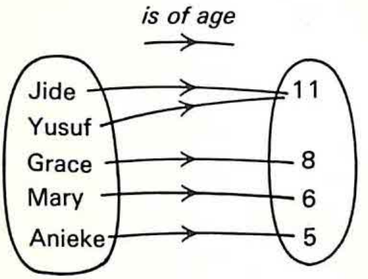
14. Draw an arrow graph for {2, 4, 6, 8, 12} to show *is a multiple of*.

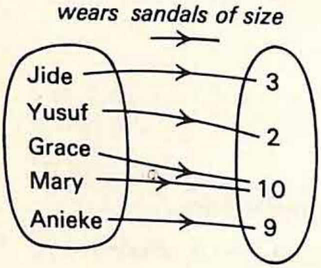
15. Draw an arrow graph for {1, 5, 10, 15, 20} to show *is a multiple of*.

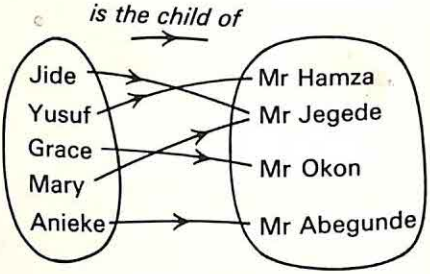
16. Choose six of your friends. Find their ages. Use the information to draw an arrow graph for *is the same age as*.

B. Relationships between the members of two sets









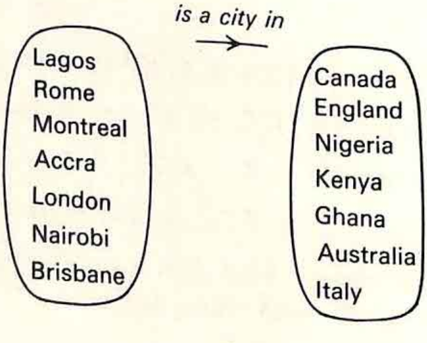
Exercises

17. Look at the four arrow graphs shown above. Write all you can about: (a) Jide, (b) Grace, (c) Anieke.

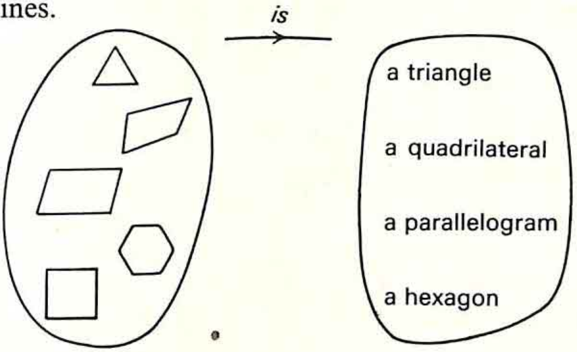
18. Are there any brothers or sisters among the five children? If so, who are they?

19. Choose five children, of different ages, and draw four arrow graphs like those above.

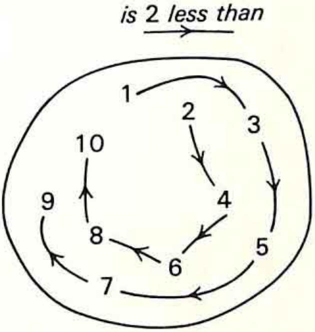
20. Copy and complete the arrow graph:

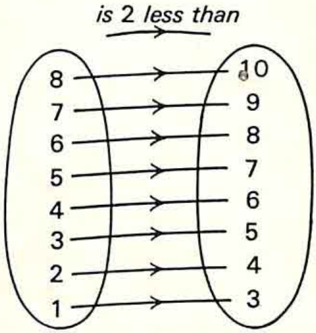


21. Copy and complete the arrow graph. Make sure you draw *all* the arrowed lines.



Look at these two arrow graphs.



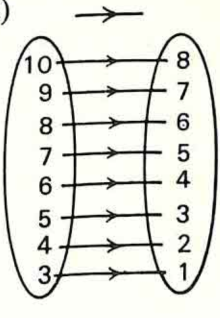


Both graphs show the same information. The second graph, however, is easier to draw and to read. For this reason the second type of arrow graph is frequently used instead of the first when we are dealing with numbers.

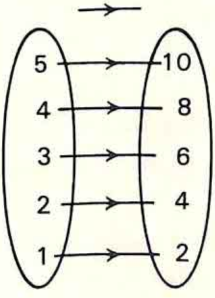
Exercises

22. What can be written for the arrowed line?

(a)



(b)



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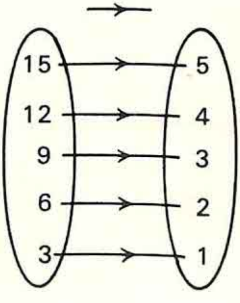
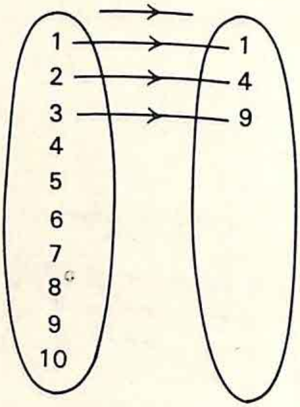


23. Draw graphs like those in exercise 22 to show:
    (a) *is a third of* for {1, 2, 3, 4} as the first set.
    (b) *is one more than twice* for {1, 3, 5, 7, 9} as the first set.
    (c) *is one less than twice* for {1, 3, 5, 7, 9} as the first set.

24. Decide what the arrowed line represents. Then copy and complete the arrow graph:

(a)



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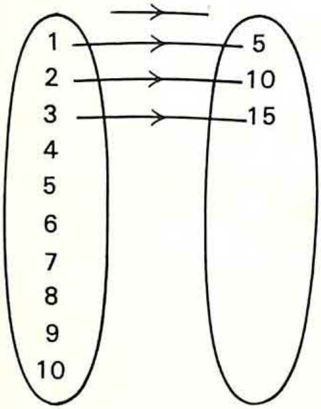

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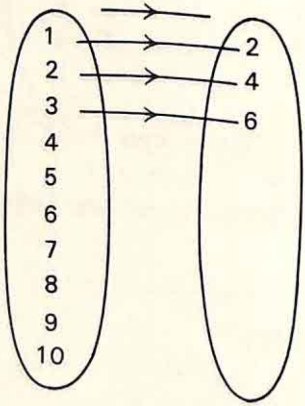
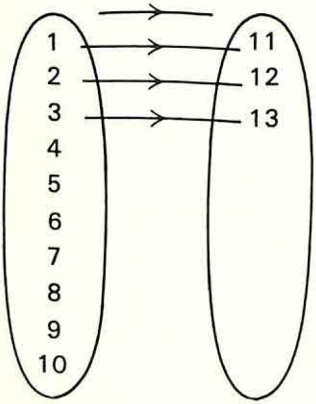


(d)



Important ideas in this chapter

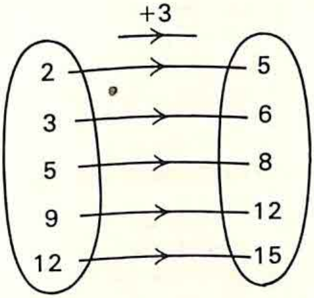
1. The use of arrow graphs to show relationships between the members of a set.
2. The use of arrow graphs to show relationships between the members of one set and those of a second set.

2. SHOWING OPERATIONS

A. Another use of an arrowed line

In Book 2, *Operations with Sets and Numbers*, we introduced the word *operation* to describe activities such as addition, multiplication, subtraction, and division. We used the terms *addition operation*, *subtraction operation*, etc. We recorded the results of the operations as $4 + 2 = 6$, $5 - 2 = 3$, etc.

We now look at a way of recording these operations in a graphical form. Here is an example:

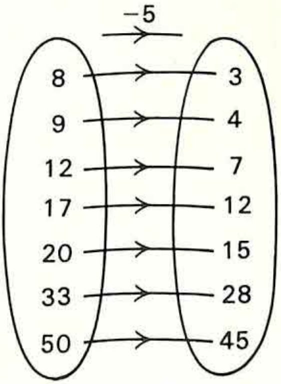


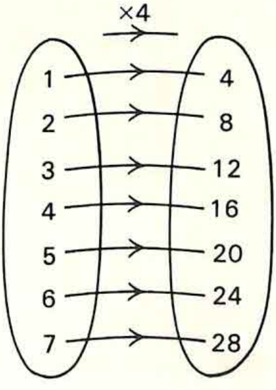
The arrowed line now represents an *instruction*. It tells us to *add three* to each member of the first set.

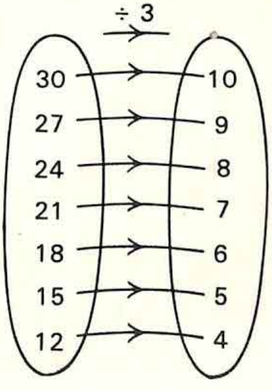
Up to now we have not used an arrowed line in this way. We have used it to show the middle part of a statement. For example, we have shown 'Jide *is taller than* Yusuf' by Jide → Yusuf.

So now we have two ways in which we use an arrowed line. This is why it is important that we should always show what the arrowed line represents on *all* the arrow graphs we draw.

Here are some more arrow graphs in which the arrowed line is used to represent an instruction.



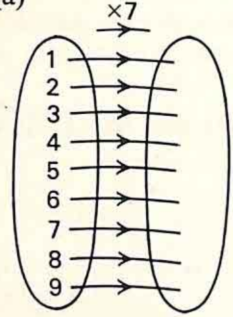




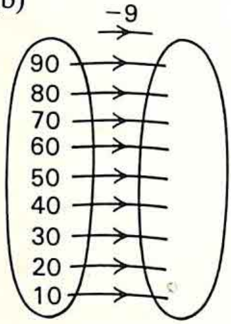
Exercises

1. Copy and complete the arrow graph:

(a)



(b)



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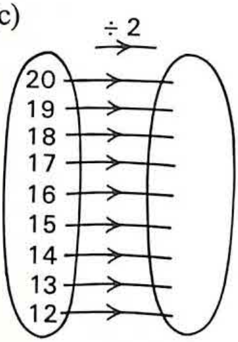
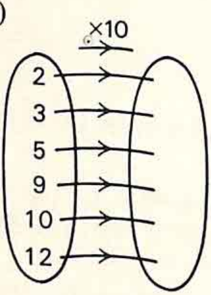


2. Copy and complete the arrow graph:

(a)



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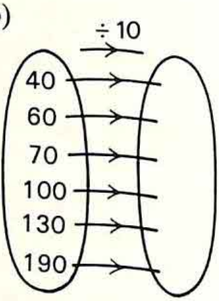

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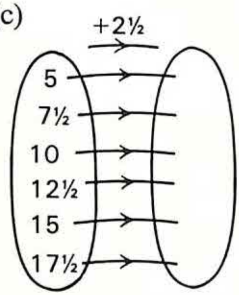
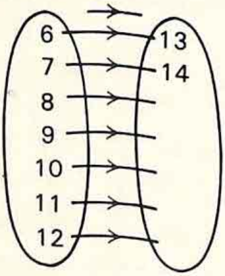


3. Copy and complete the arrow graph:

(a)



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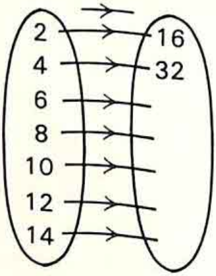

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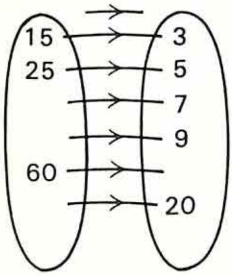


4. Draw an arrow graph for:

(a) {6, 8, 10, 12, 14, 16} and $\xrightarrow{\div 4}$.

(b) {2, 12, 22, 32, 42, 52, 62} and $\xrightarrow{+10}$.

B. The idea of an operator

When we use × 7 as an instruction to multiply each member of a set by seven, we call it an **operator**

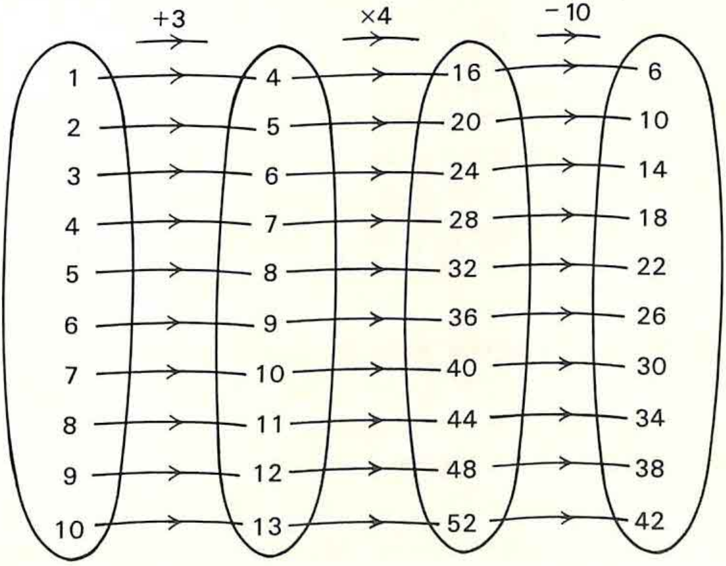
Used in the same way, $+3$, -5, $\div 8$ are other examples of operators. An operator acts on each member of a set and produces a second set. The idea of an operator is very important. It underlies many of the operations done by a computer.

Of course, there is nothing new in the idea of an operator except the name. We have often used the idea. For example:

$6 + 2$ can be read as 6 *add 2.*
$6 - 2$ can be read as 6 *subtract 2.*
6×2 can be read as 6 *multiply by 2.*
$6 \div 2$ can be read as 6 *divide by 2.*

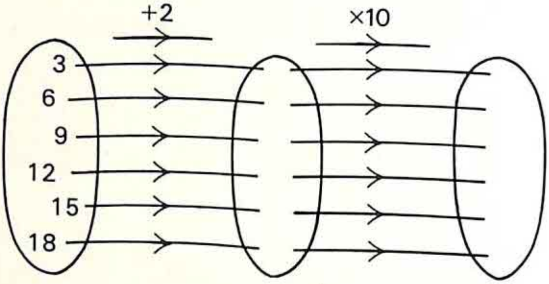
The difference is that instead of thinking of two numbers with an operation (e.g. 6, 2, and addition), we tend to think of one number (e.g. 6) with an operator (e.g. $+2$). The result in each case is 8. With a computer or desk calculator, we put the first number in the machine. We then press a key to add the second number.

We can have more than one operator acting on a set of numbers. They can, of course, only act one at a time. Here is an example.

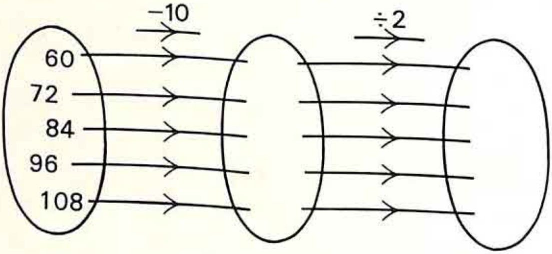


Exercises

5. Copy and complete the arrow graph:



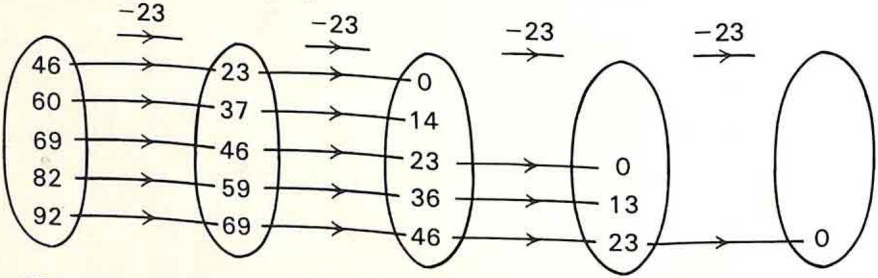
6. Copy and complete the arrow graph:



7. Make up an arrow graph for the set of operators (in the order given):

(a) {+3, ×2, −5} (b) {×3, +10, −7} (c) {×2, ×2, ×2}

8. Look at this arrow graph.



Use it to give the answers to the following divisions:

(a) 46 ÷ 23 (b) 60 ÷ 23 (c) 69 ÷ 23 (d) 82 ÷ 23 (e) 92 ÷ 23

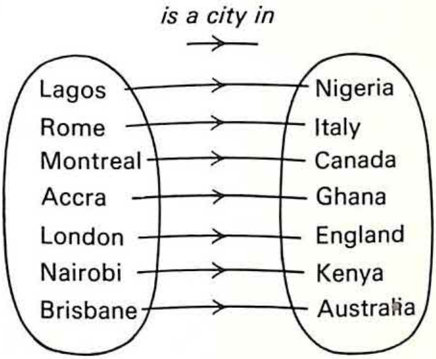
Important ideas in this chapter

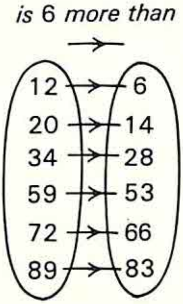
1. The idea of an operator.
2. The use of arrow graphs to show operations on sets of numbers.

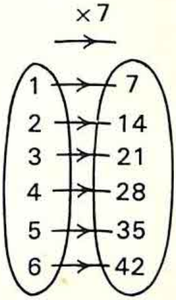
3. RELATIONS

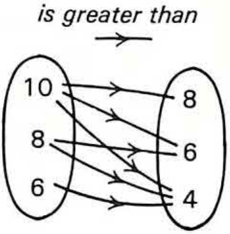
A. Ordered pairs

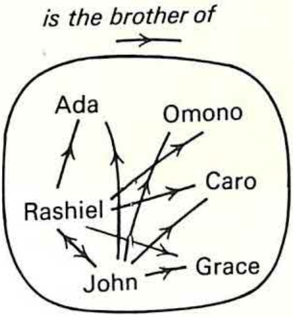
Look at these arrow graphs.

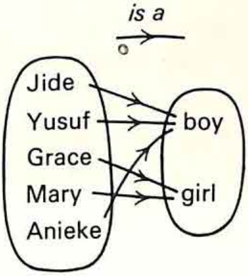












The first arrow graph shows the set of statements:

Lagos *is a city in* Nigeria.
Rome *is a city in* Italy.
Montreal *is a city in* Canada.
Accra *is a city in* Ghana.
London *is a city in* England.
Nairobi *is a city in* Kenya.
Brisbane *is a city in* Australia.

There is another way in which this set of statements can be shown in a shorter form. The cities and the corresponding countries are shown as: {(Lagos, Nigeria), (Rome, Italy), (Montreal, Canada), (Accra, Ghana), (London, England), (Nairobi, Kenya), (Brisbane, Australia)}

In the same way the set of statements for the second arrow graph on page 13 are shown as:

{(12, 6), (20, 14), (34, 28), (59, 53), (72, 66), (89, 83)}

Exercise

1. Show, in the new way, the set of statements for each of the other four arrow graphs on page 13.

We have to be very careful, of course, to make sure that the two members of each pair are written in the right order. For example,

20 *is 6 more than* 14 must be shown as (20,14).

It would be wrong to show it as (14,20) as 14 is not 6 more than 20. Likewise,

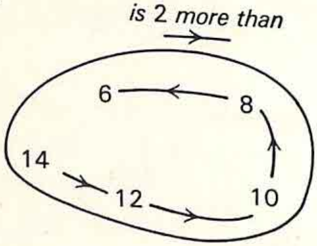
Lagos *is a city in* Nigeria must be shown as (Lagos, Nigeria).

Because the order in which the two members are written is important each of the pairs written above is called an **ordered pair**. The idea of an ordered pair is very important. As we progress in mathematics we shall find that it is increasingly used.

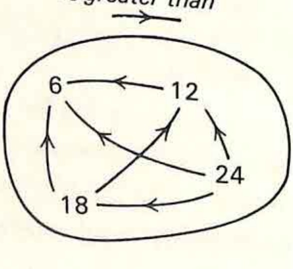
Exercises

2. Show the information in the arrow graph as a set of ordered pairs.

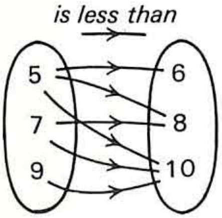
(a)



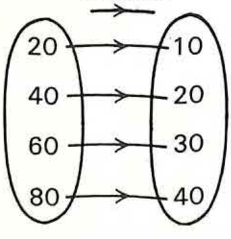
(b)



(c)

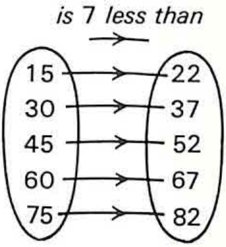


(d)

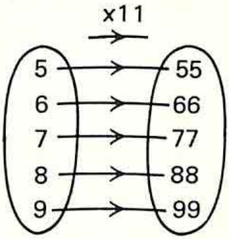


3. Write down the set of ordered pairs shown in the arrow graph.

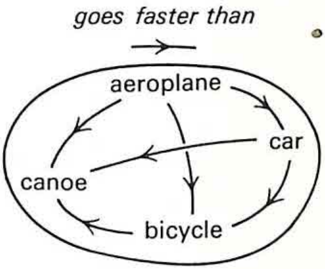
(a)



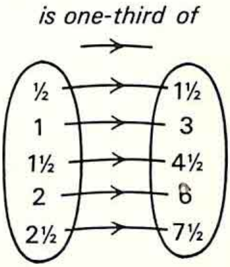
(c)



(b)



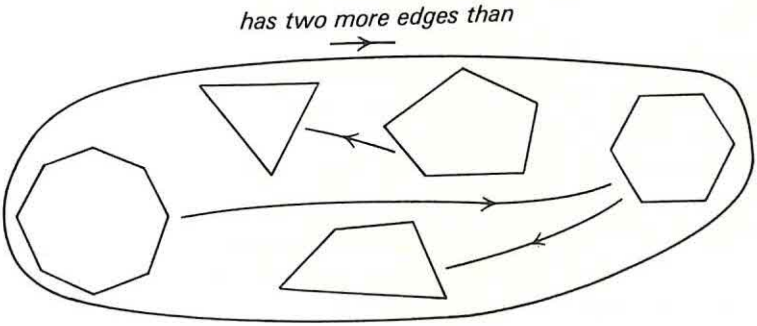
(d)



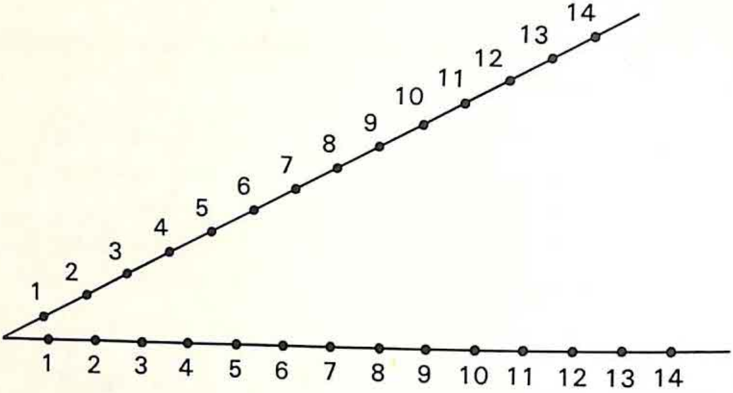
4. Draw an arrow graph to show the set of ordered pairs. Show what the arrowed line represents.

(a) {(12, 6), (10, 5), (8, 4), (6, 3), (4, 2), (2, 1)}
(b) {(hexagon, square), (hexagon, triangle), (square, triangle)}
(c) {(6, 5), (6, 4), (6, 3), (5, 4), (5, 3), (4, 3)}
(d) {(7, 10), (7, 9), (7, 8), (8, 10), (8, 9), (9, 10)}
(e) {(1, 4), (2, 8), (3, 12), (4, 16), (5, 20), (6, 24)}
(f) {(100, 10), (90, 9), (80, 8), (70, 7), (60, 6), (50, 5), (40, 4)}

5. Show this arrow graph as a set of ordered pairs:

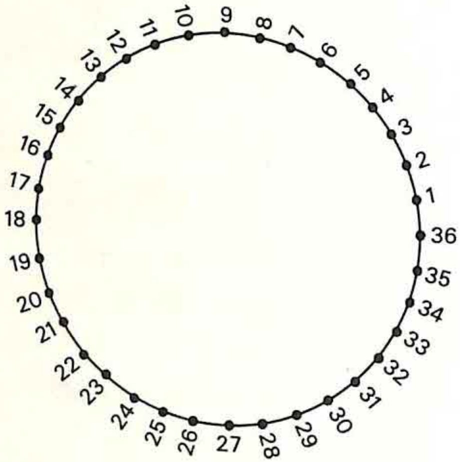


6.



Draw and mark two lines as shown.
Draw a straight line from the point marked 1 on one line to the point marked 14 on the other. Call this line (1, 14).
Go on to draw the lines: (2, 13), (3, 12), (4, 11), (5, 10), (6, 9), (7, 8), (8, 7), (9, 6), (10, 5), (11, 4), (12, 3), (13, 2), (14, 1).

7.

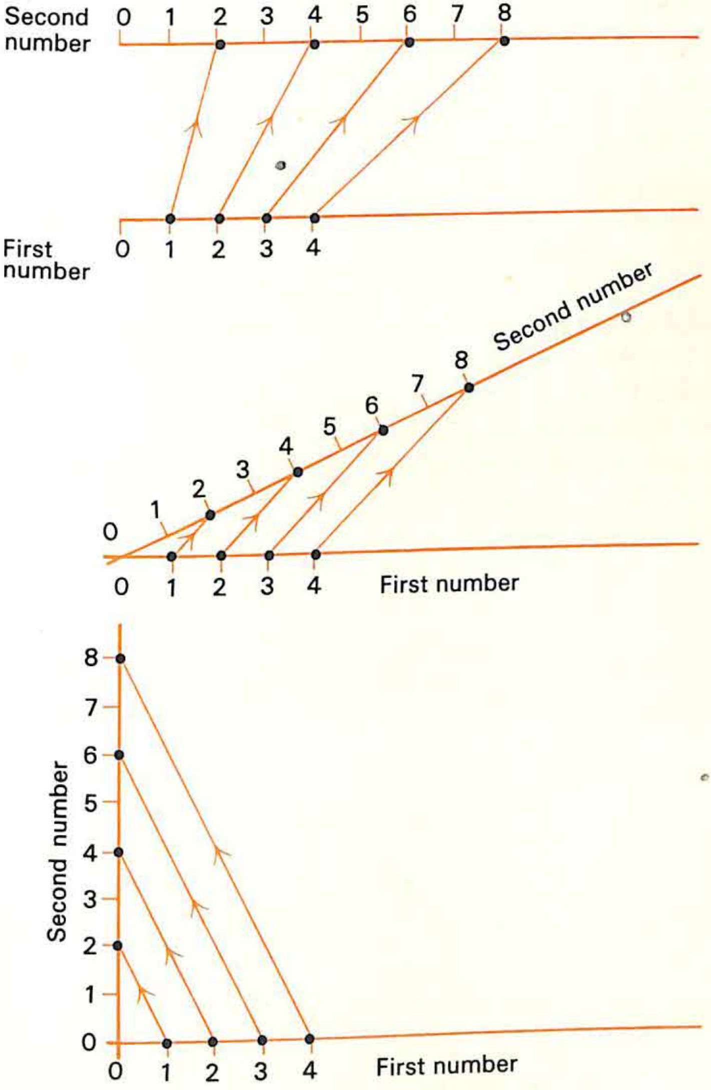


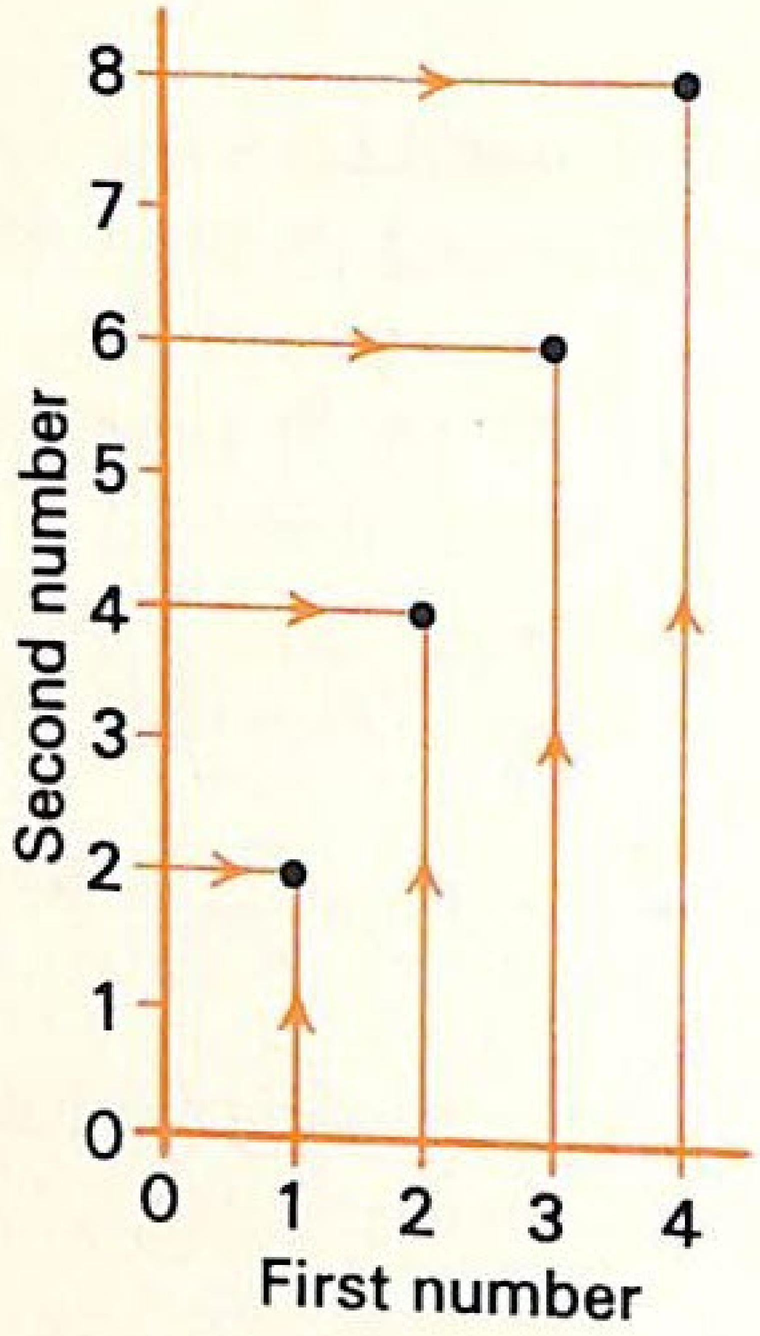
Draw a large circle and mark it as shown.
Draw the lines: (1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12), . . . , . . . , . . . , (16, 32), (17, 34), (18, 36).
The next ordered pair is (19, 38). But there is no point marked 38. Going round the circle, however, 38 is 2 more than 36. So we think of (19, 38) as (19, 2) and draw a line from point 19 to point 2.
Go on in this way and draw lines for: (20, 40), (21, 42), . . . , . . . , (35, 70), (36, 72).

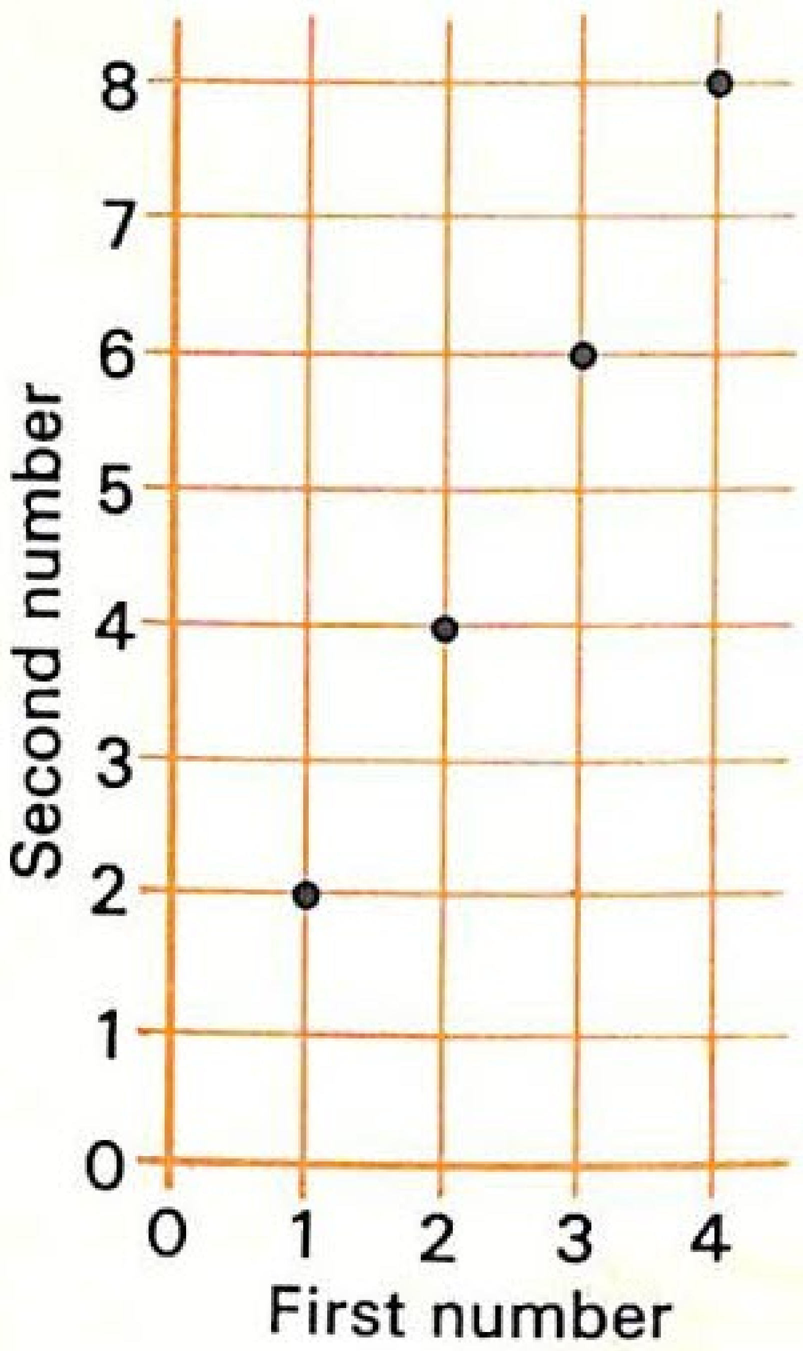
B. Showing ordered pairs on a grid

In the earlier books in this series we showed numbers as points on a number line. We now look at some ways of showing pairs of numbers, using number lines.

Here are several ways of showing $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$.



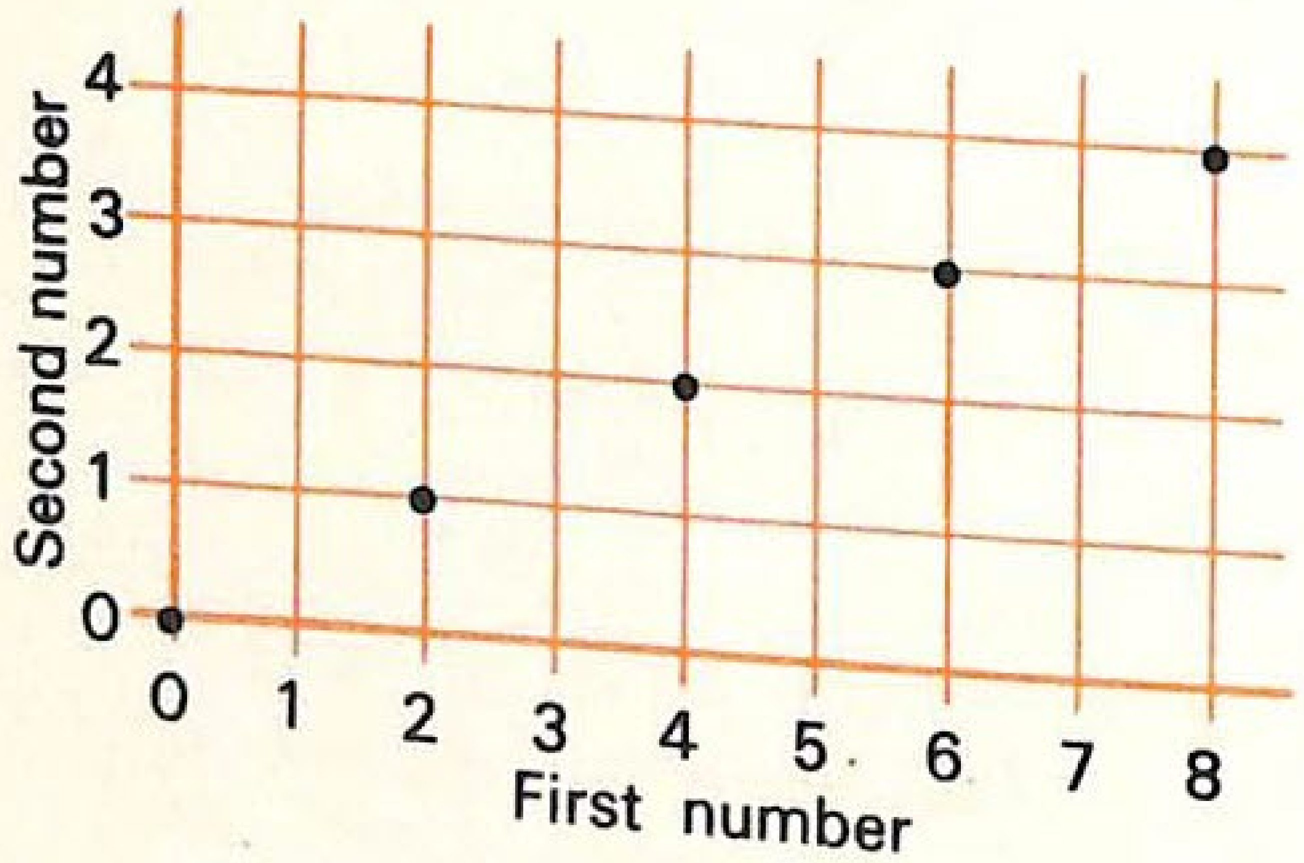




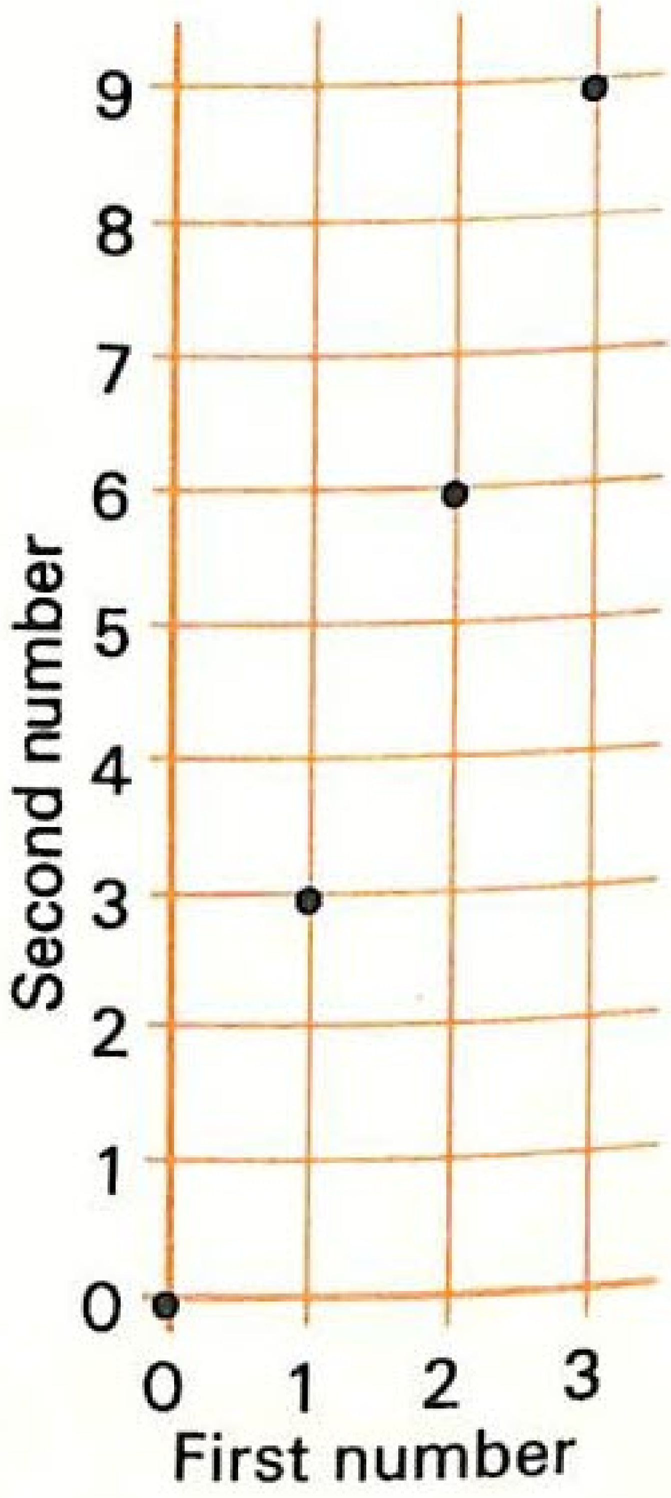
Each of the five diagrams shows the set of ordered pairs. Any of them can be used for this purpose but the one most frequently used is that shown above, on the right. If a prepared grid is used, the points showing the ordered pairs can be quickly marked.

Here are two other examples of ordered pairs shown as points on a grid.

{(0, 0), (2, 1), (4, 2), (6, 3), (8, 4)}



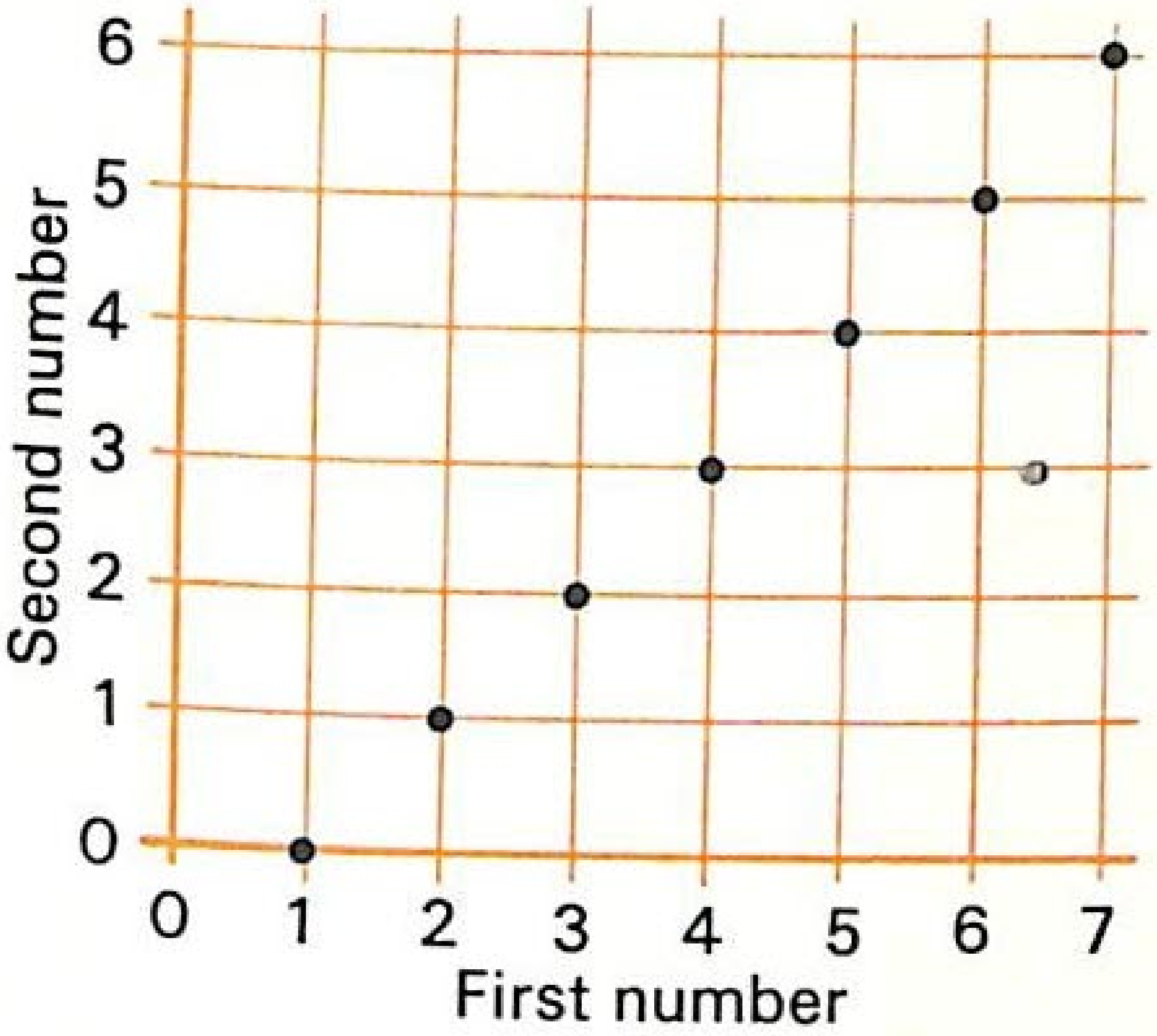
{(0, 0), (1, 3), (2, 6), (3, 9)}



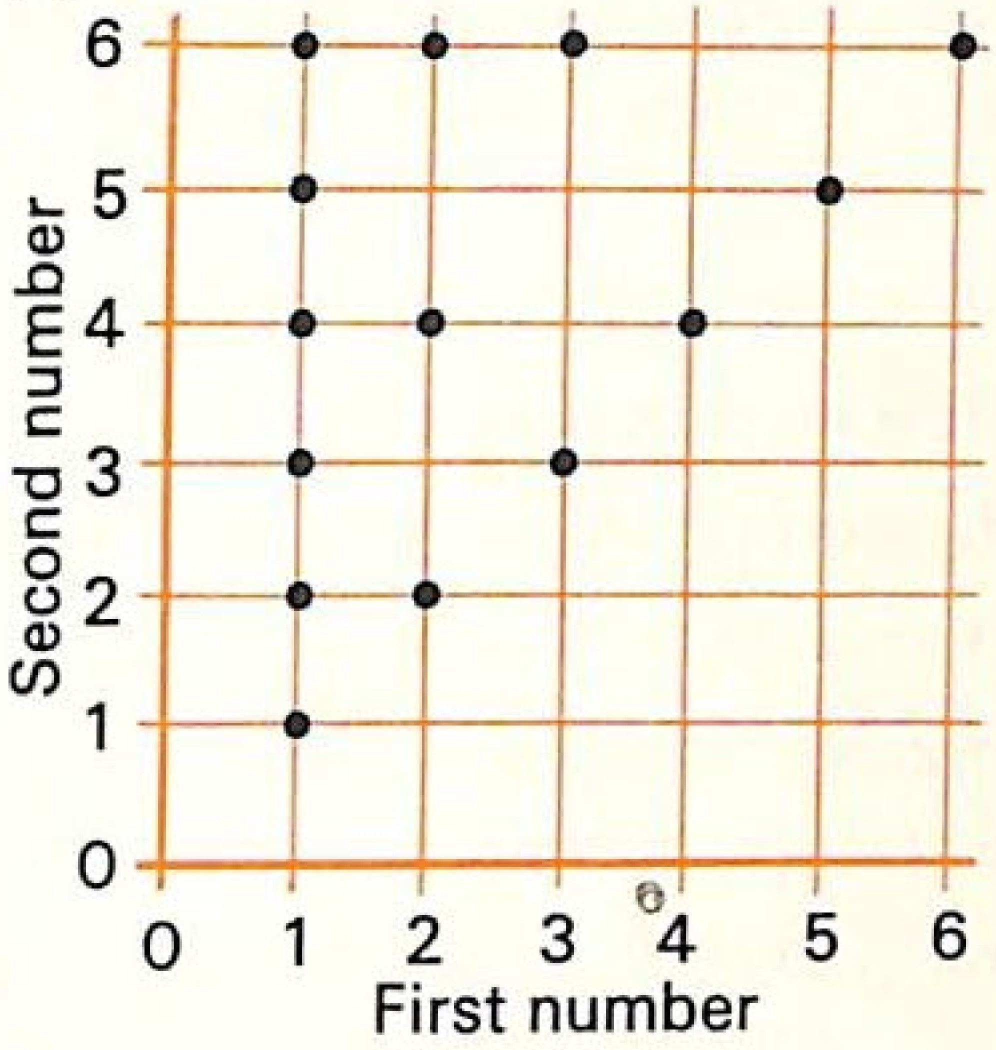
Exercises

8. (i) Write the set of ordered pairs shown on the grid.
   (ii) Show the ordered pairs by an arrow graph.
   (iii) What does the arrowed line represent in the arrow graph?

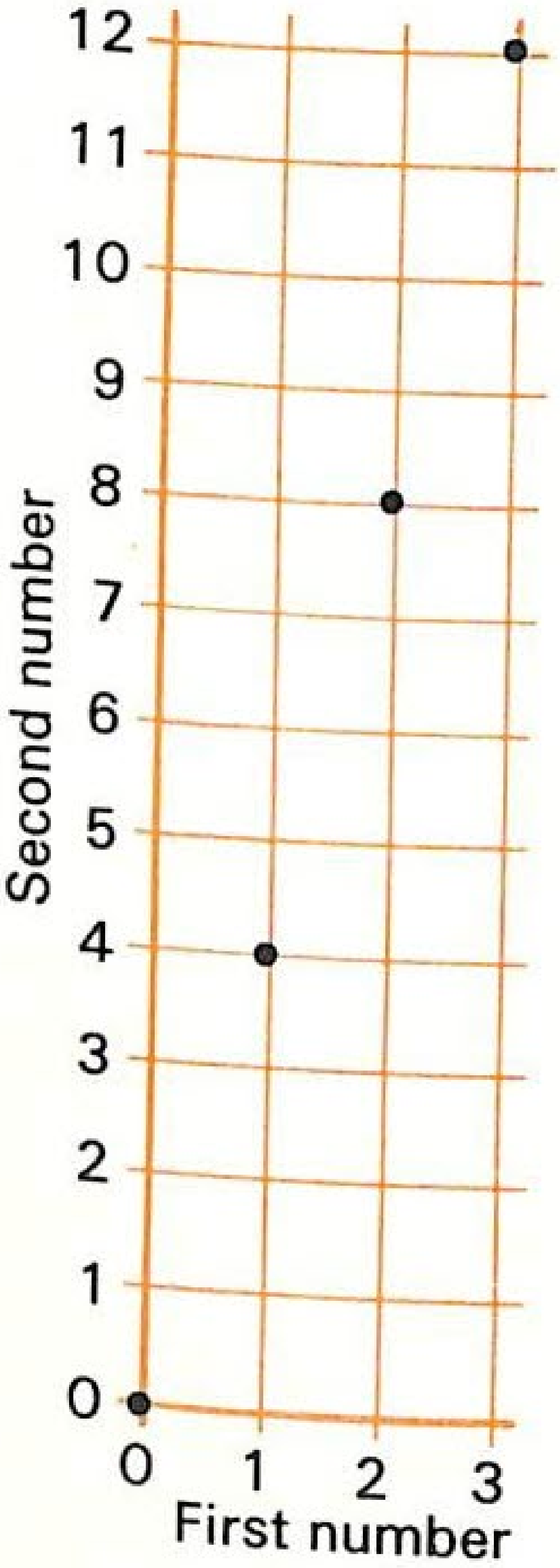
(a)



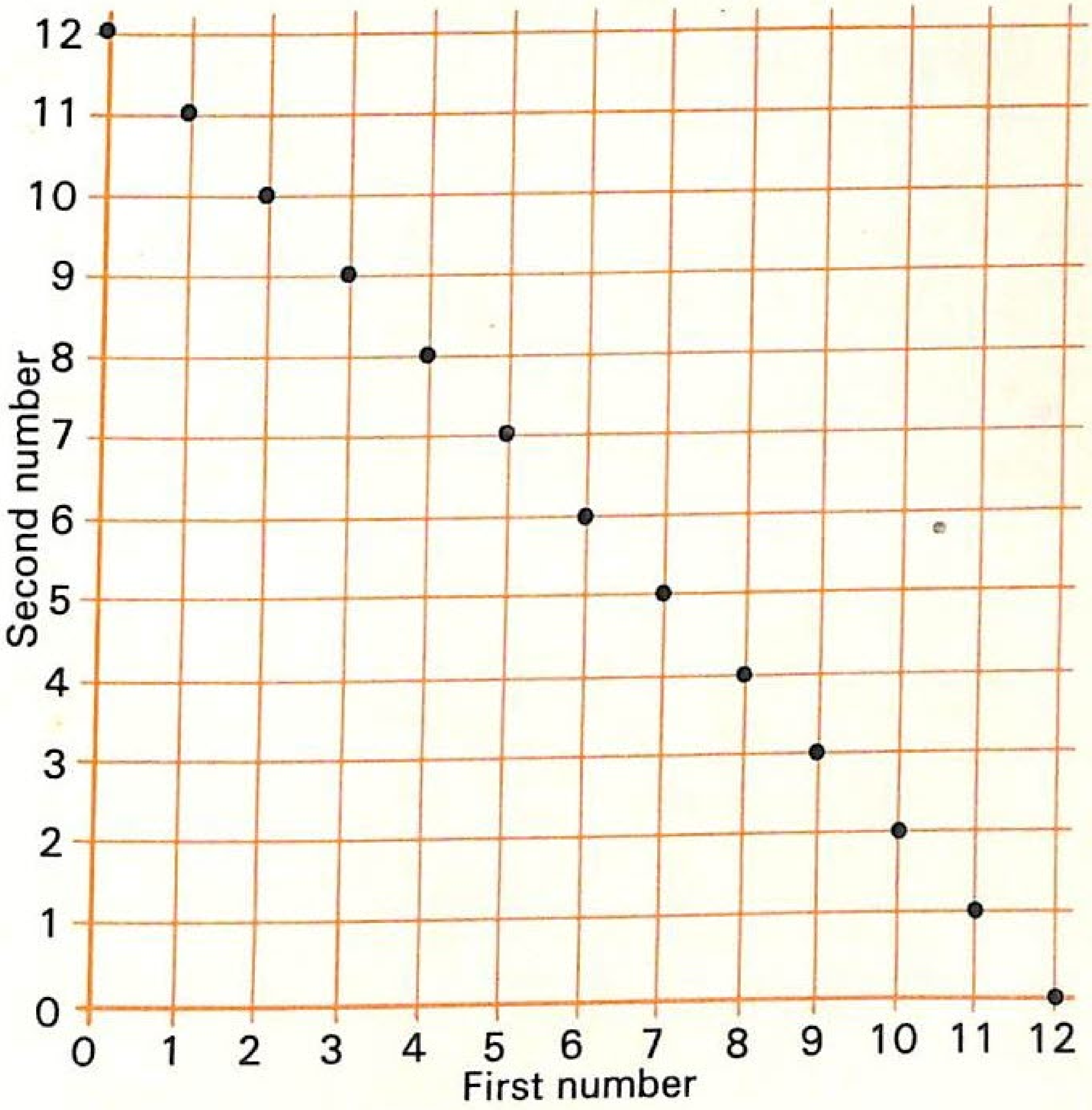
(c)



(b)



(d)



9. Show the set of ordered pairs on a grid:

(a) {(0, 0), (3, 1), (6, 2), (9, 3)}

(b) {(6, 1), (6, 2), (6, 3), (6, 6), (5, 1), (5, 5), (4, 1), (4, 2), (4, 4), (3, 1), (3, 3), (2, 1), (2, 2), (1, 1)}

(c) {(1, 0), (3, 1), (5, 2), (7, 3), (9, 4), (11, 5)}

For each write a statement to show how the second number of each pair is related to the first.

In exercise 8(a) the set of first numbers is {1, 2, 3, 4, 5, 6, 7} and the set of second numbers is {0, 1, 2, 3, 4, 5, 6}.

In exercise 8(b) the set of first numbers is {0, 1, 2, 3} and the set of second numbers is {0, 4, 8, 12}.

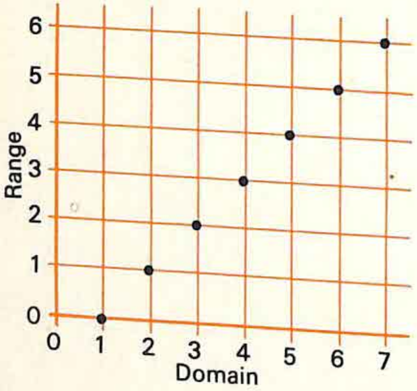
Special names are given to these two sets. The set of first numbers is called the **domain**. The set of second numbers is called the **range**

So in exercise 9(a) the domain is {0, 3, 6, 9}
and the range is {0, 1, 2, 3}.

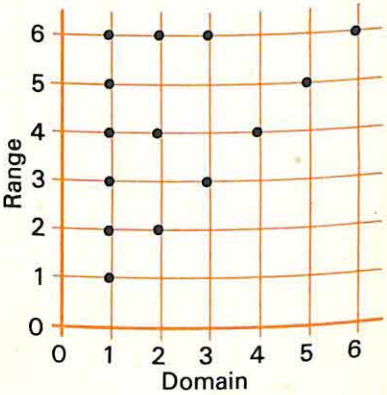
In exercise 8(c) the domain is {1, 2, 3, 4, 5, 6}
and the range is {1, 2, 3, 4, 5, 6}.

Using domain and range, exercises 8(a) and 8(c) are shown as:

(a)



(c)



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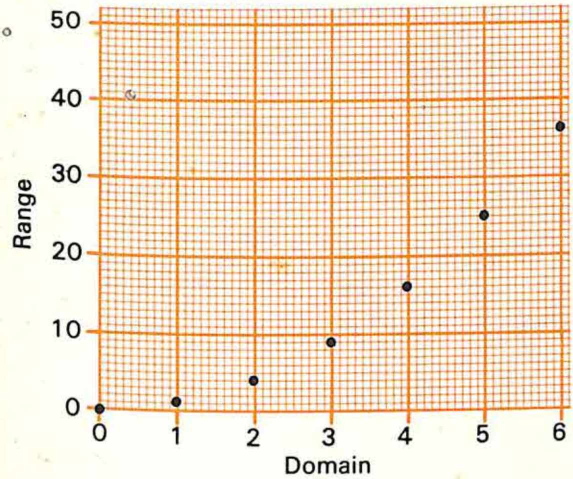
The members of the domain and range are not always numbers. For example in the first arrow graph on page 13,
the domain is
{Lagos, Rome, Montreal, Accra, London, Nairobi, Brisbane}
the range is
{Nigeria, Italy, Canada, Ghana, England, Kenya, Australia}.
In the last arrow graph on the same page,
the domain is {Jide, Yusuf, Grace, Mary, Anieke}
the range is {boy, girl}.

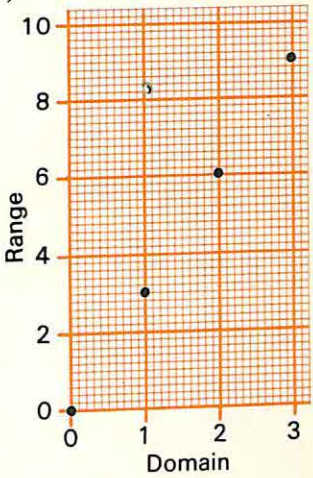
Exercises

10. Write the domain and the range.

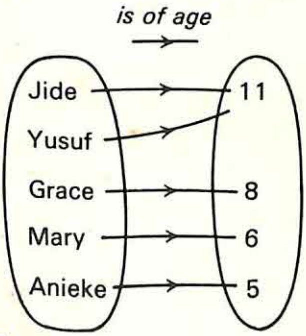
(a)



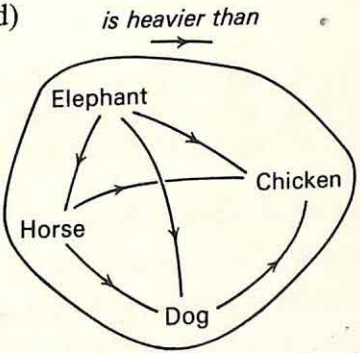
(c)



(b)



(d)



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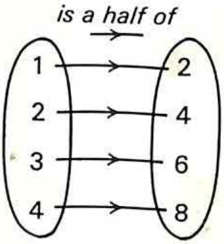
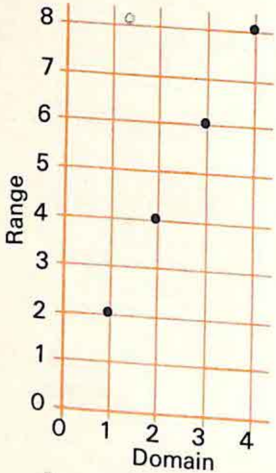
C. The idea of a relation

In each of the exercises we have done so far we have been showing the connection between the members of a set or between the members of one set and those of a second set. The result has been shown as a set of statements, an arrow graph, as points on a grid, or as a set of ordered pairs. For example:

1 *is a half of* 2
2 *is a half of* 4
3 *is a half of* 6
4 *is a half of* 8





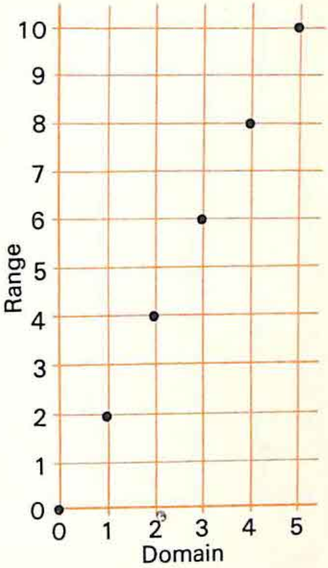
$\{(1, 2), (2, 4), (3, 6), (4, 8)\}$

The set of statements, in whatever form it is shown, is called a **relation**. Any of the forms can be used but as we go on in mathematics we shall find that it is best to think of a relation as a set of ordered pairs.

Important ideas in this chapter

1. The idea of ordered pairs.
2. Showing ordered pairs on a grid.
3. The idea of a relation.

4. USING LETTERS TO SHOW RELATIONS



Look at this relation.
The set of ordered pairs is:
{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)}.

The domain is {0, 1, 2, 3, 4, 5}.

The range is {0, 2, 4, 6, 8, 10}.

The connection between the first number and the second number of each ordered pair can be expressed as:

either

first number *is a half of* second number

or

second number = (first number) × 2.

Instead of writing 'first number' and 'second number' in full we can save time by using:

f for 'first number' and s for 'second number'

The connection between the two numbers of each pair is then shown as:

either $f = \frac{1}{2} s$ *or* $s = f \times 2$.

The second of the two ways can be written as $s = 2 \times f$, or more shortly, as $s = 2f$

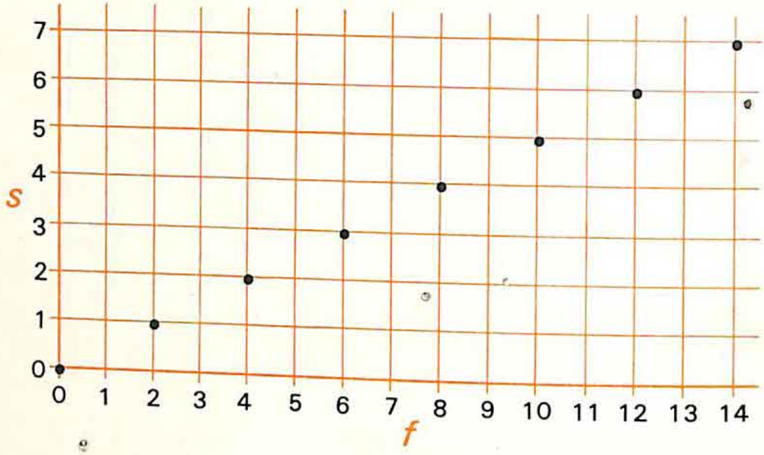
Exercises

1. Use s and f to show the connection between the first number and the second number of each ordered pair in the relation:

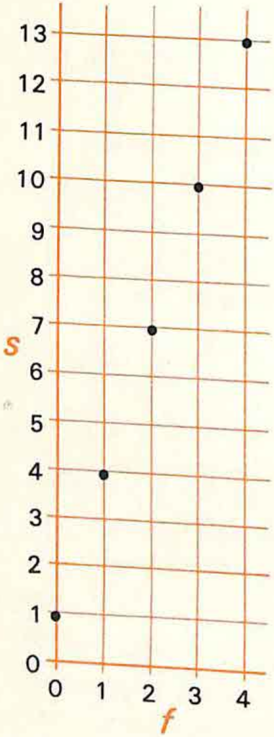
   (a) {(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21)}
   (b) {(1, 2), (2, 3), (3, 4), (4, 5)}
   (c) {(1, 0), (2, 1), (3, 2), (4, 3), (5, 4), (6, 5)}
   (d) {(0, 0), (2, 1), (4, 2), (6, 3), (8, 4)}
   (e) {(0, 1), (1, 3), (2, 5), (3, 7), (4, 9), (5, 11)}

2. Use *f* and *s* to describe the relation represented by the set of dots.

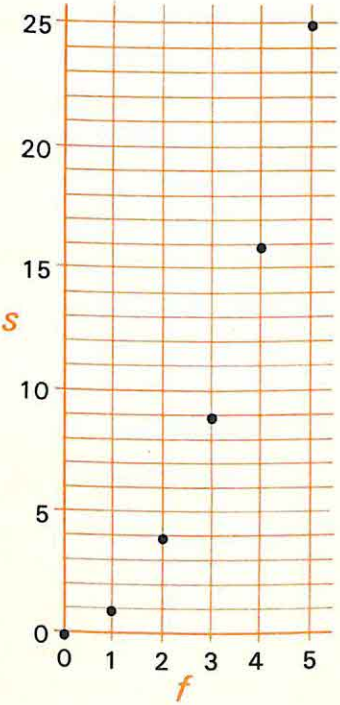
(a)



(b)



(c)



3. Show on a grid the relation represented by:

(a) $s = f + 2$ (for $f = 1, f = 2, f = 3, f = 4, f = 5$).
(b) $s = 2f - 3$ (for $f = 2, f = 3, f = 4, f = 5, f = 6$).

In exercise 3(a) the values of f to be used can be shown more shortly in several ways. For example:

$$0 < f < 6 \qquad 0 < f \leqslant 5 \qquad 1 \leqslant f \leqslant 5$$

The first way, $0 < f < 6$, can be read as:
nought *is less than* f and f *is less than* six.
It could also be read as:
f *is between* nought and six.

Note: (i) At this stage we are using only natural numbers for f. Later we shall use fractions.
(ii) When we say that f *is between* nought and six, we imply that *all* the natural numbers between 0 and 6 are used, in turn, for f.

The second way, $0 < f \leqslant 5$, can be read as:
nought *is less than* f and f *is less than or equal to* five.

The third way, $1 \leqslant f \leqslant 5$, can be read as:
one *is less than or equal to* f and f *is less than or equal to* five.

Each of these three ways describes the set $\{1, 2, 3, 4, 5\}$. Using, for example, the third way, we can now rewrite (a) and (b) of exercise 3 as:

(a) $s = f + 2 \quad (1 \leqslant f \leqslant 5)$
(b) $s = 2f - 3 \quad (2 \leqslant f \leqslant 6)$

Note: In the above section we have used *is less than or equal to* and the symbol $\leqslant$. We could, of course, use *is greater than or equal to* and the symbol $\geqslant$, to show the values of f. That is, $5 \geqslant f \geqslant 1$.

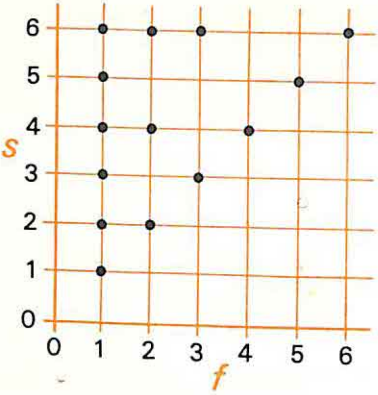
Exercise

4. Show in a shorter way:

(a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
(b) $\{0, 1, 2, 3, 4, 5, 6\}$
(c) $\{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
(d) $\{95, 96, 97, 98, 99, 100, 101, 102\}$

USING LETTERS TO SHOW RELATIONS

It is not always possible to use f and s to show a relation. For example, in the graph showing *is a factor of* for $\{1, 2, 3, 4, 5, 6\}$, we see there is more than one value of s for $f = 1$. This is also true for $f = 2$ and $f = 3$. In this case it is not possible to show the connection between s and f in a simple way.

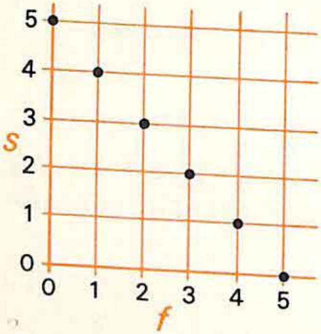


It is much easier to show the connection between s and f when to each value of f there is only one value of s. When there is only one value of s for each value of f the relation is given a special name. It is called a **function**

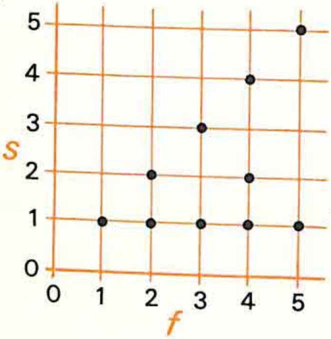
Exercises

5. Is the relation a function?

(a)



(b)



6. Is the relation a function?

   (a) $\{(0, 1), (1, 4), (2, 7), (3, 10)\}$
   (b) $\{(1, 2), (1, 3), (1, 4)\}$
   (c) $\{(1, 3), (2, 1), (2, 5), (3, 0), (3, 6)\}$
   (d) $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$

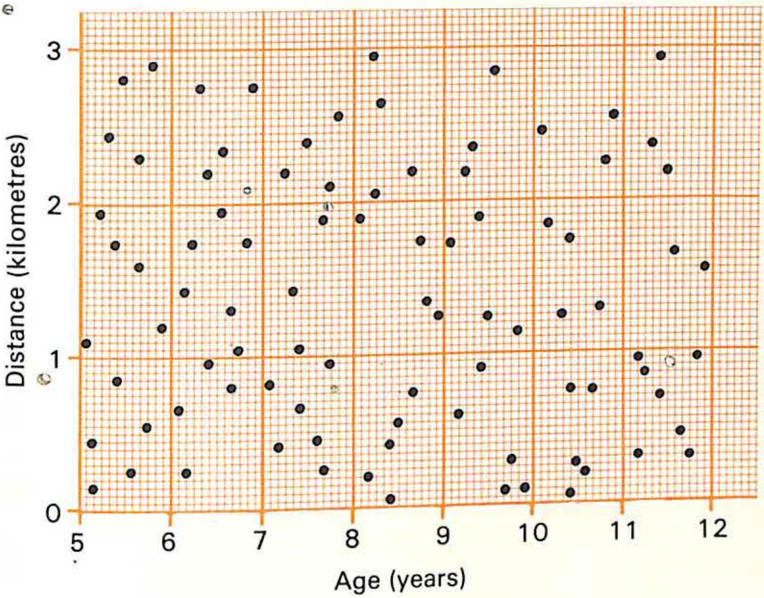
Important ideas in this chapter

1. The use of letters to show a relation.
2. The use of $<$, $>$, $\leqslant$, and $\geqslant$ in describing a set of numbers.
3. The idea of a function.

5. CONTINUOUS LINE GRAPHS

A. The use of a scatter graph

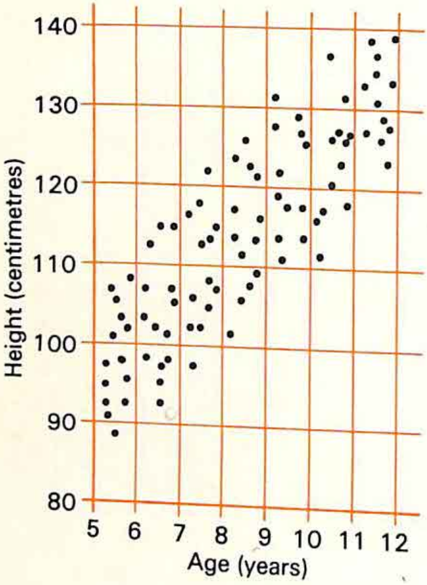
A teacher in a school decided to find out how far the children lived from school. He found the distances for ninety children of various ages. The results are shown below. Each dot shows the information for one child.



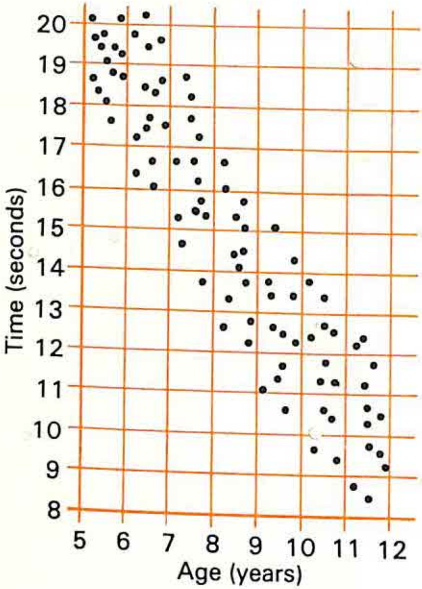
Exercises

1. From the graph find how many children:
   - (a) aged 8 to 9 live less than 2 km from the school.
   - (b) aged 6 to 7 live less than 1 km from the school.
   - (c) aged 9 to 10 live less than $\frac{1}{2}$ km from the school.
   - (d) aged 11 to 12 live more than 2 km from the school.
2. From the graph find how many children altogether live:
   - (a) more than 2 km from the school.
   - (b) less than $2\frac{1}{2}$ km from the school.
3. What fraction of the children live:
   - (a) within $\frac{1}{2}$ km from the school?
   - (b) more than $1\frac{1}{2}$ km from the school?
   - (c) less than $1\frac{1}{2}$ km from the school?

A graph to show the height (in centimetres) of each of the ninety children.



A graph to show the time taken by each of the ninety children to run 50 metres.



Exercises

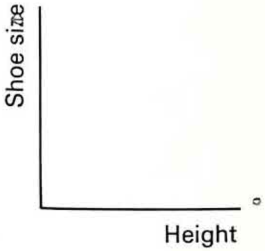
4. Look at the two graphs above. What do you notice?
   In what ways do the two graphs differ?
   In what ways do the graphs differ from the graph on page 27?
5. For each child in a set of 30 (or more) children record his age, his height, and the time he takes to run 60 metres.
   Show the results as graphs, like those above.
6. Weigh each child in the set of exercise 5.
   Draw a graph to show how their masses are related to their ages.

Each of the three graphs shown above is called a **scatter graph**

Exercises

7. Make a drawing to show what you think the pattern of dots would look like if you drew a scatter graph for:

(a)



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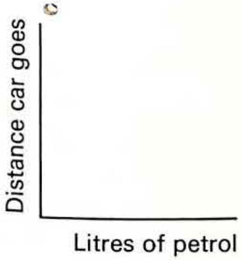

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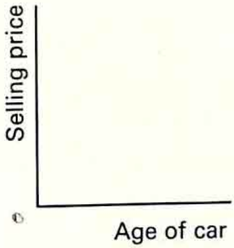

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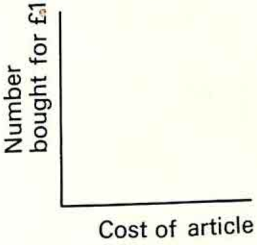

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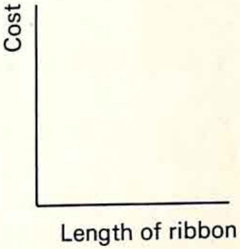

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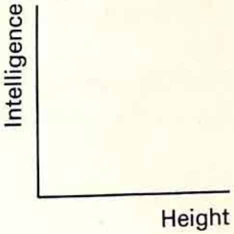


8. Record in a table the number of pages and the cost of each of thirty, or more, books.
Show your results as a scatter graph.
What does your graph tell you about the relationship between the number of pages and the cost of a book?

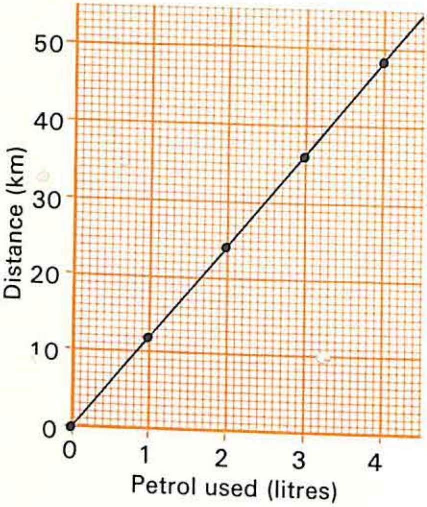
9. Record in a table the length and the width of different rectangles with a perimeter of 48 cm.
Show your results as a scatter graph. What do you notice?

10. Suspend a length of elastic (a rubber band or a light spring will do equally well) from a nail. Attach various masses (e.g. 50 grammes, 100 grammes, etc.) to the lower end of the elastic and for each measure the total length of the stretched elastic.
Show your results as a scatter graph. What do you notice?

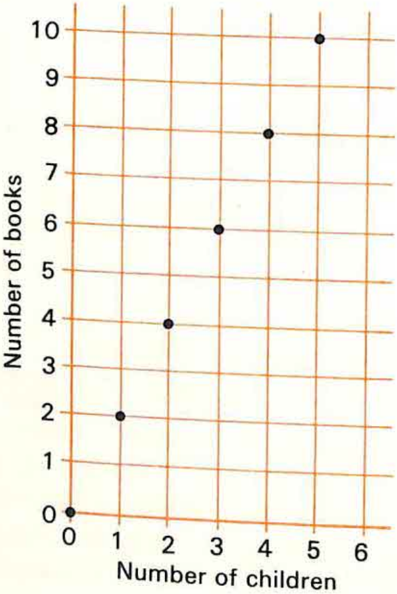
B. Joining points on a graph

In many of the graphs we have drawn most of us have noticed that the points seemed to be in a straight line or in a curve. Some of us may have thought of joining the points to make the straight line or the curve. Let us find out whether we ought to do so.

A car goes 12 km on 1 litre of petrol.
The points on the graph show how far the car goes on various numbers of litres of petrol.
These points are all on a line.
If we consider another point on the line (e.g. that for 1½ litres) we see that the corresponding distance is what we would expect (i.e. 18 km).
This is true for all points on the line.
So in this example we are right to draw a line through the points.

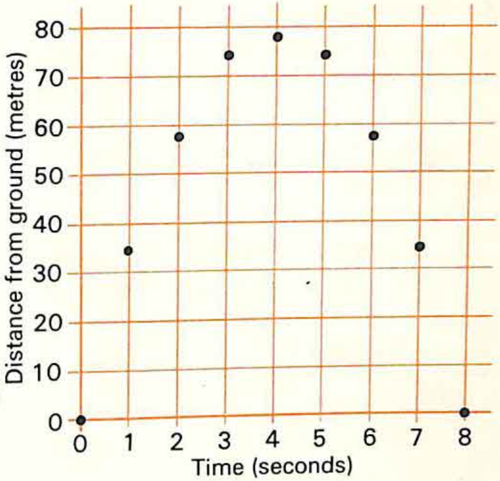


This graph shows the number of books needed to provide two each for various numbers of children.
Again we could draw a straight line to pass through all the points.
It would, however, serve no purpose. We can have only whole numbers of children and whole numbers of books.
It would be wasting our time to say, for example, that 1¼ children need 2½ books.
It would be wrong to draw a line in this case.

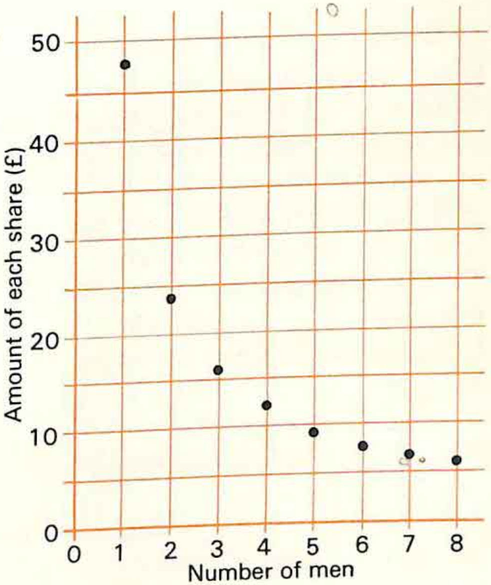


Exercises

11. A metal ball is shot upwards and its distance from the ground is measured every second. The results are shown in the graph. Would it be right to join the marked points by a curved line? Give reasons for your answer.



12. Forty-eight pounds are shared equally among some men. The graph shows how much each man gets according to the number of men sharing. Would it be right to join the marked points by a curved line? Give reasons for your answer.



From the four graphs we have just considered we realise that we have to think very carefully before we join the marked points on a graph. Sometimes it is sensible and correct to do so. The in-between points represent possible ordered pairs. For other sets of points it is wrong to join them. The in-between points have no meaning.

C. Naming lines

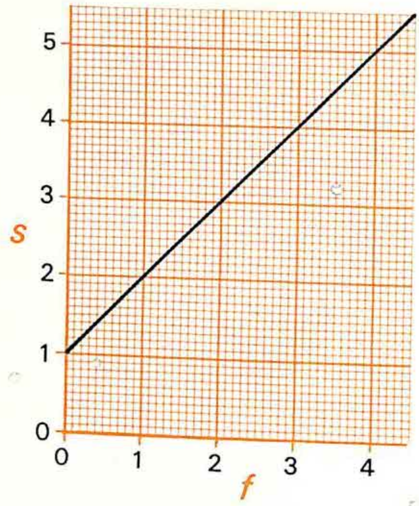
If we look very carefully at the line we see that for every point on it

$$s = f + 1$$

For this reason we often refer to the line itself as

the line $s = f + 1$

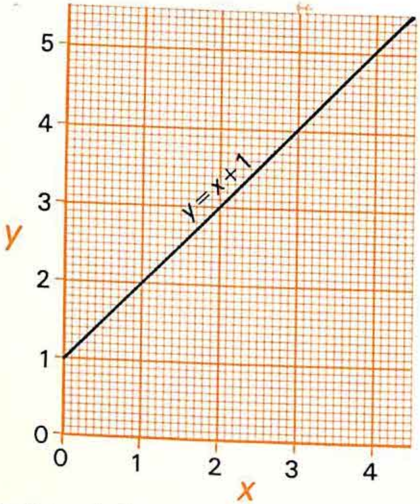
This is a helpful notation as it tells us the relationship between the two numbers of the ordered pair shown by each point on the line.



Although f and s are the two letters which we first thought of using to represent *first* and *second*, they are not the ones usually used for this purpose. The letter x is usually used for the first number, and the letter y for the second number. There is no particular reason for this choice. It is just tradition. Using x and y we refer to the line as

$$y = x + 1$$

This statement is often called the **equation** of the line.



The line shown is for values of x from 0 to 4. It could, of course, be extended for larger domains.

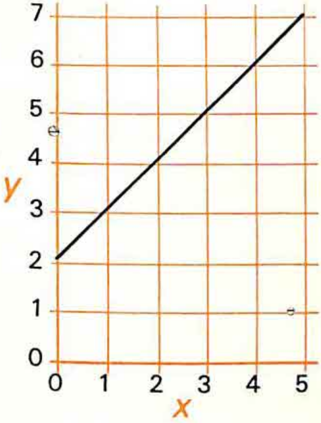
Exercises

13. Draw the line:

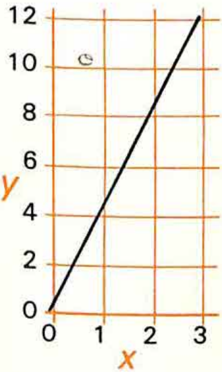
(a) $y = 2x$ for $0 \leqslant x \leqslant 5$
(b) $y = 2x - 1$ for $1 \leqslant x \leqslant 6$
(c) $y = \frac{1}{2}x$ for $0 \leqslant x \leqslant 10$
(d) $y = 10 - x$ for $0 \leqslant x \leqslant 10$
(e) $y = 5x - 4$ for $1 \leqslant x \leqslant 4$
(f) $y = 6 - \frac{1}{3}x$ for $0 \leqslant x \leqslant 9$

14. Find the equation of the straight line. What is the domain?

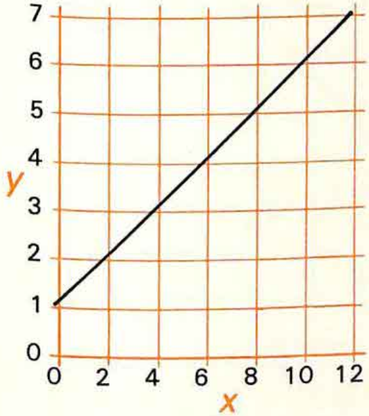
(a)



(b)



(c)



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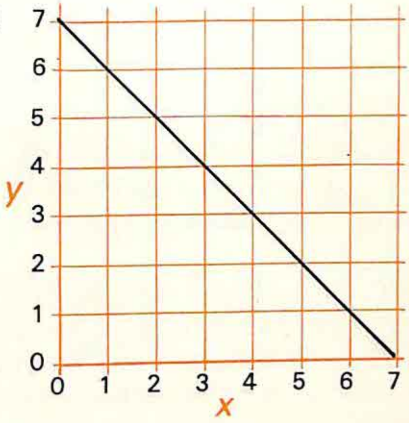

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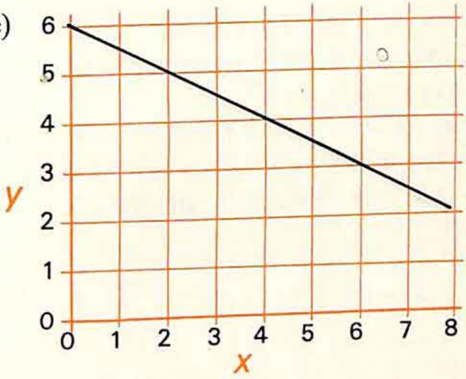
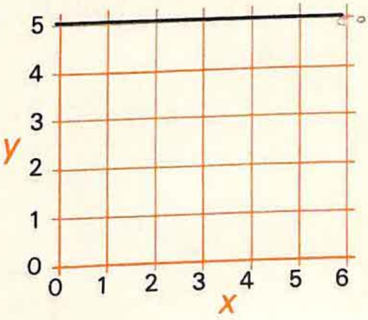


(f)



15. A set of squares have edges of length 1 cm, 2 cm, 3 cm, . . . , 10 cm. Find the area of each square. Show the results in a table, like this:

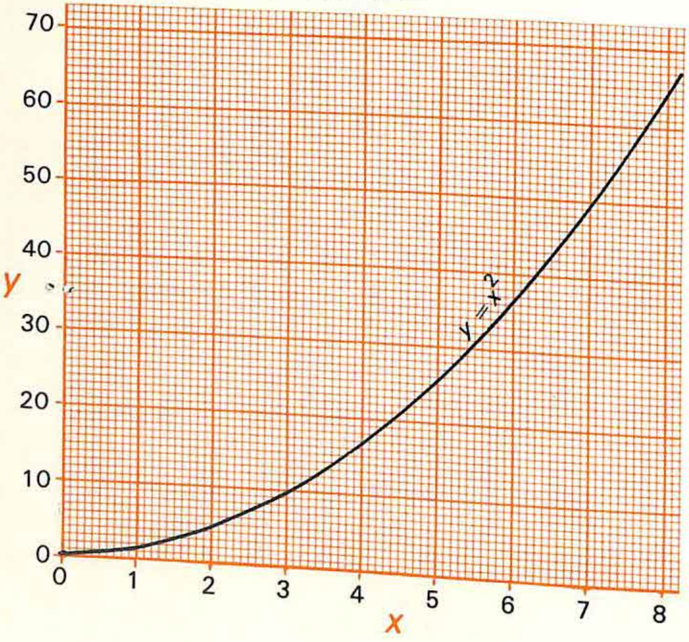
Length of edge (cm)	1	2	3	4	5	6	7	8	9	10
Area of square (cm²)										

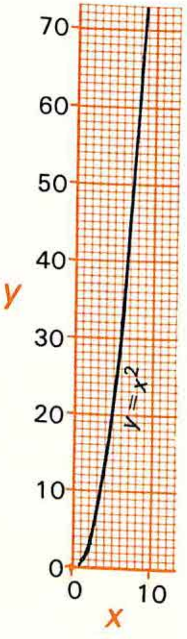
Also show the results as a graph.
Do you think it is right to join the points? If so, say why.
What is the equation of the line?

In exercise 15 most of us probably drew a curved line, part of which is shown below. In the graph on the left convenient scales have been used for x and y. In the graph on the right the same scale has been used for both x and y. We also probably wrote down the equation correctly as $y = x \times x$.

There is, however, another way of writing $x \times x$. We can write it as x^2. We read this as *x squared*. Using this notation we write the equation of the curve as $y = x^2$.

This curve is called a **parabola**. We always get a parabola when we show graphically a set of ordered pairs in which the second number of each pair is the square of the first.

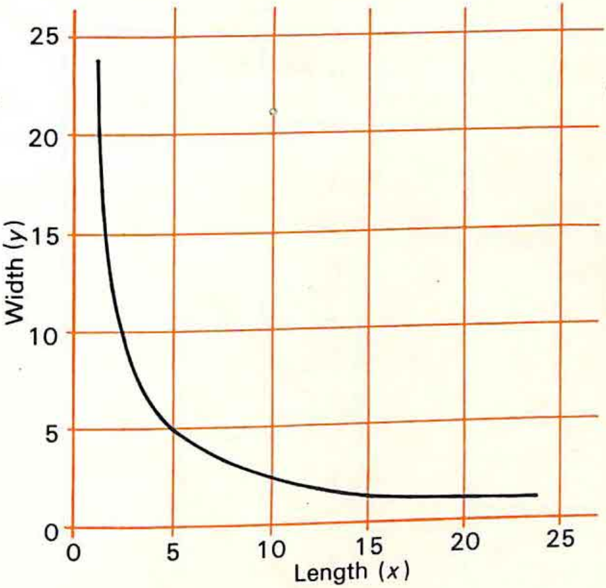




Exercise

16. A rectangular piece of tin is to be cut with an area of 24 cm^2. Write, in table form, some of the possible lengths and widths of the rectangle.
    Show your results as a graph.
    Do you think it is right to join the points? If so, say why.
    What is the equation of the line?

In exercise 16 most of us probably decided that we could join the marked points as the length of the rectangle could be any number of centimetres between 0 and 24. It did not have to be a whole number. The kind of curved line obtained is shown below.



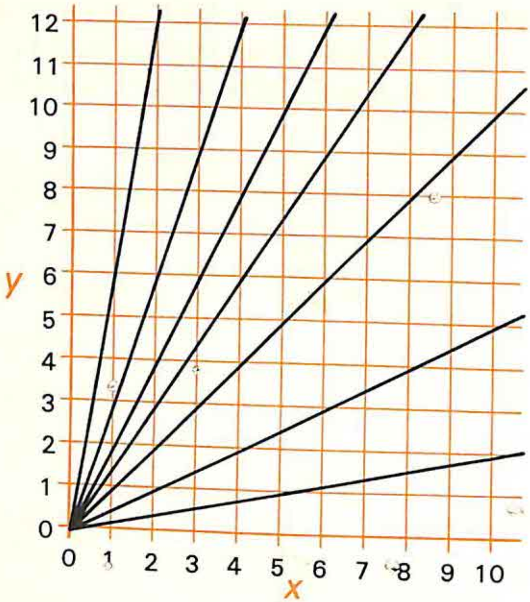
The equation of the line can be written in various ways. For example:

$$x \times y = 24 \qquad xy = 24 \qquad y = 24 \div x \qquad y = \frac{24}{x}$$

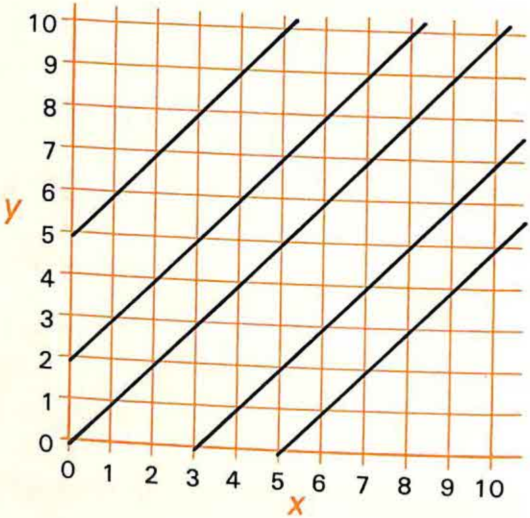
This is another important curve. It is called a **hyperbola**. The graph of a set of ordered pairs for which the product of each pair of numbers is the same is always a hyperbola.

Exercises

17. Copy the graph. Write the equation of each line. What do you notice? How does the equation of a line indicate the steepness of the line?



18. Copy the graph. Write the equation of each line. What do you notice?



19. How would you decide whether two straight lines are parallel by looking at their equations?

20. A man starts to work at a salary of £700 a year. Each year he has an increase of £20.
Draw a graph to show how much he earns each year during his first ten years.

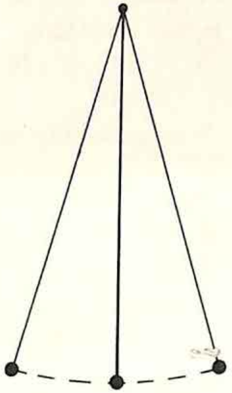
21. A car travels 12 kilometres on 1 litre of petrol.
Draw a graph to show how far it travels as it uses each of the 6 litres in its tank.

22. Use the graph on page 34 to find the area of a square with edge:
(a) 6·5 cm (b) 3·5 cm (c) 4·5 cm

23. Use the graph on page 34 to find the edge of a square with area:
(a) 50 cm² (b) 30 cm² (c) 60 cm²

24. Collect a set of circular objects (tins, plates, wheels, etc.). Measure the diameter and circumference of each.
Show your results as a graph.
What do you notice?

25. Tie a small loop at one end of a piece of fine string about 60 cm long. Put the loop over a nail in a wall.
At the bottom end of the string tie a small piece of metal.
Measure the distance from the nail to the piece of metal.
Now let the metal swing to and fro through a *small* angle.
Measure the time, in seconds, for 20 swings.
Now change the length of the string.
Again find the time for 20 swings.
Repeat for other lengths of the string (from 30 cm to 100 cm).
Show your results as a graph.
Can you decide from your graph how long the string must be so that a swing takes one second?



26. Measure, to the nearest centimetre, the heights and chest measurements of 50 boys of about the same age.
Show the results as a graph. What do you notice?

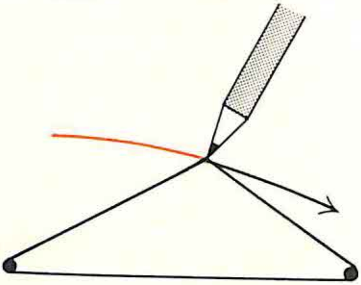
D. Drawing some interesting curves

We finish this chapter by looking at some of the ways in which we can draw several of the curves which occur in nature and in the world around us.

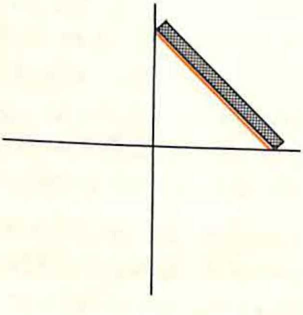
Exercises

27. This is an activity we use in marking out a football pitch. Put a stake in the ground. Tie a loop at one end of a length of strong rope or thin wire and place it round the stake. At the other end of the rope fasten a paint brush.
    Keeping the rope taut and brush vertical (and dipped into paint from time to time) move forward and paint the ground.
    What shape is obtained?
    Indoors, on a sheet of paper, a drawing pin can replace the stake, a piece of string can be used in place of the rope, and a pencil in place of the brush.
    A pair of compasses is based on the same idea.

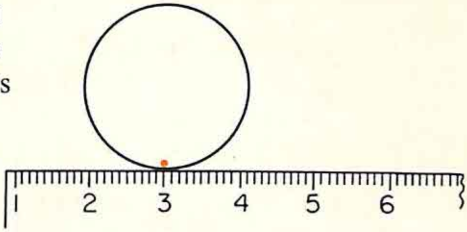
28. Place a sheet of plain paper on a drawing board or other flat wooden surface.
    Place two pins 10 cm apart in the board.
    Arrange a loop of string, about 25 cm long, round the pins and place a pencil in the loop, as shown.
    Keeping the string taut, move the pencil.
    What shape is traced out?
    Investigate the effect of varying the distance between the two pins.
    What relationship between an ellipse and a circle is indicated?

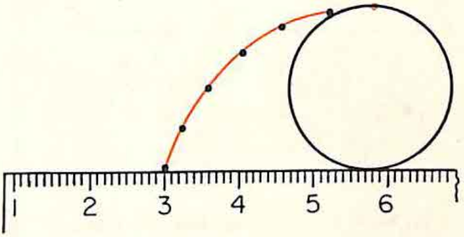


29. Draw two lines at right-angles to each other. Place a strip of wood or thick card in the position shown.
    Draw the coloured line.
    Move the strip to other positions (making sure its ends touch the two lines). Repeat many times (in all the four spaces).
    An astroid is produced.

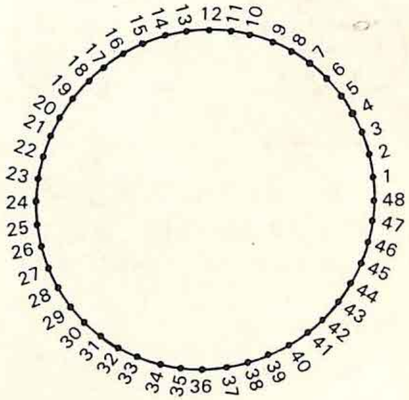


30. Draw a straight line on a sheet of paper and place a large coin on the paper and on the line, as shown.
    Place the edge of a ruler along the line and ask a friend to hold it in place for you.
    Mark the coin where it touches the line.
    Slowly roll the coin along the ruler and from time to time mark, on the paper, the position of the red mark.
    Join these marked points.
    A cycloid is obtained.





31. Draw a large circle and mark it off into 48 equal parts as shown.
    Number the points from 1 to 48.
    Join, by straight lines, the pairs of points: (1, 2), (2, 5), (3, 8), (4, 11), (5, 14), etc.
    Draw a line *from* every marked point.
    Remember, for example, that (20, 59) can be thought of as (20, 11). (59 is once round the circle and 11 more.)
    A curve called a nephroid is obtained.
    Note: Instead of joining the points by pencil, coloured thread can be used. This produces a colourful design.



Important ideas in this chapter

1. The use of a scatter graph to show relations.
2. Deciding when points on a graph can be joined.
3. The idea of the equation of a line.
4. Practical ways of drawing curves.

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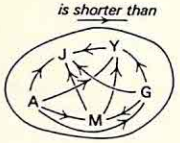
ANSWERS

1. SHOWING RELATIONSHIPS

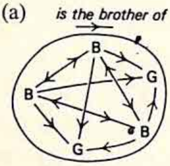
1. (a) Jide (b) Anieke
Jide is the tallest; Anieke is the shortest.

2.



3. (a) Rashiel (b) Ada

4. (a)



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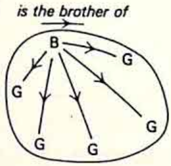
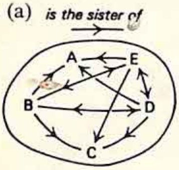


5. (a) Two; A and C. (b) Three; K. S, and J.

6. (a)



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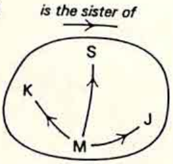

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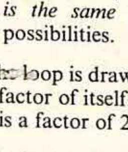

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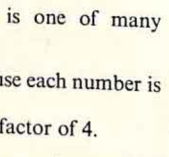
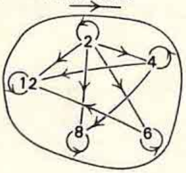


9. (a) *is one third of.*
(b) *is the same age as* is one of many possibilities.

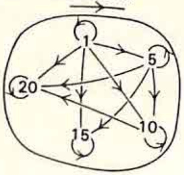
10. The loop is drawn because each number is a factor of itself.
2 is a factor of 2; 4 is a factor of 4.

11. The loop is drawn because each number is a multiple of itself. 3 is a multiple of 3.

12.



13.



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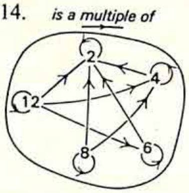

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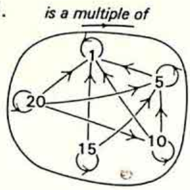
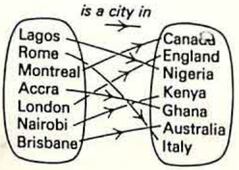


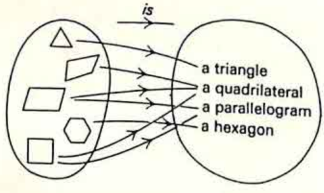
17. (a) Jide is in class 6 and is 11 years old.
His father is Mr Jegede.
Jide wears size 3 sandals.
(b) Grace is in class 4 and is 8 years old.
Her father is Mr Okon.
Grace wears size 10 sandals.
(c) Anieke is in class 1 and is 5 years old.
His father is Mr Abegunde.
Anieke wears size 9 sandals.

18. Jide and Mary are brother and sister.

20.

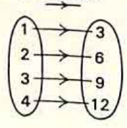


21.

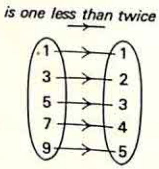


22. (a) *is 2 more than* (c) *is three times*
(b) *is a half of*

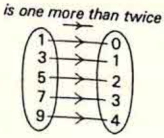
23. (a)



(c)

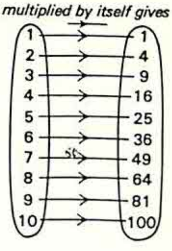


(b)

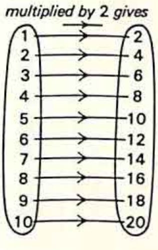


ANSWERS

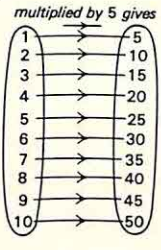
24. (a)



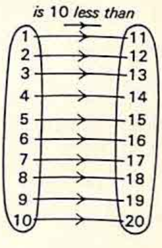
(b)



(c)

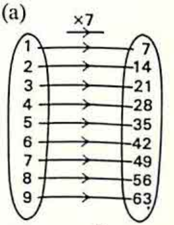


(d)

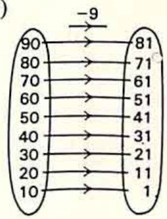


2. SHOWING OPERATIONS

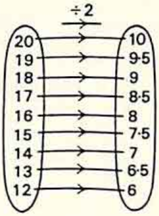
1. (a)



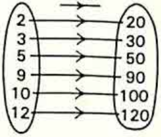
(b)



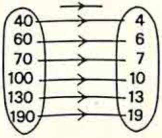
(c)



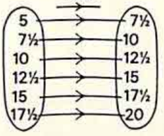
2. (a)



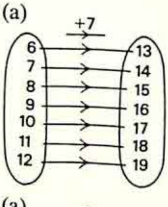
(b)



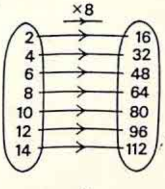
(c)



3. (a)



(b)



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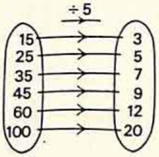
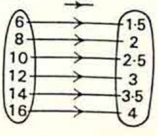
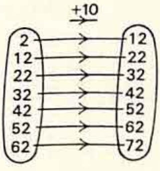


4. (a)



(b)



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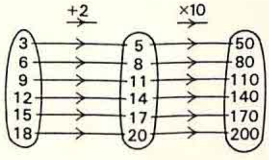

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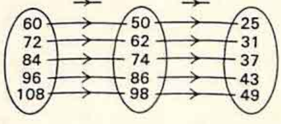


8. (a) 2 (c) 3 (e) 4
(b) 2, rem. 14 (d) 3, rem. 13

3. RELATIONS

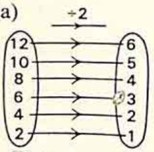
1. {(1, 7), (2, 14), (3, 21), (4, 28), (5, 35), (6, 42)}
{(10, 8), (10, 6), (10, 4), (8, 6), (8, 4), (6, 4)}
{(Rashiel, Ada), (Rashiel, Omono), (Rashiel, Caro), (Rashiel, Grace), (Rashiel, John), (John, Rashiel), (John, Ada), (John, Grace), (John, Caro), (John, Omono)}

{(Jide, boy), (Yusuf, boy), (Grace, girl), (Mary, girl), (Anieke, boy)}

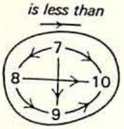
2. (a) {(14, 12), (12, 10), (10, 8), (8, 6)}
   (b) {(24, 18), (24, 12), (24, 6), (18, 12), (18, 6), (12, 6)}
   (c) {(5, 6), (5, 8), (5, 10), (7, 8), (7, 10), (9, 10)}
   (d) {(20, 10), (40, 20), (60, 30), (80, 40)}

3. (a) {(15, 22), (30, 37), (45, 52), (60, 67), (75, 82)}
   (b) {(aeroplane, car), (aeroplane, bicycle), (aeroplane, canoe), (car, bicycle), (car, canoe), (bicycle, canoe)}
   (c) {(5, 55), (6, 66), (7, 77), (8, 88), (9, 99)}
   (d) $\{(\frac{1}{2}, 1\frac{1}{2}), (1, 3), (1\frac{1}{2}, 4\frac{1}{2}), (2, 6), (2\frac{1}{2}, 7\frac{1}{2})\}$

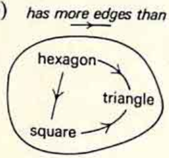
4. (a)



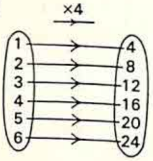
(d)



(b)



(e)



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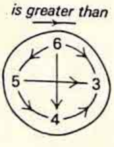
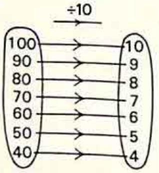
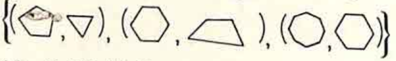


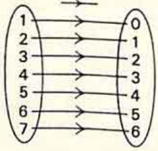
(f)



5.

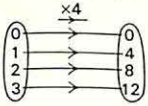


8. (a) (i) {(1, 0), (2, 1), (3, 2), (4, 3), (5, 4), (6, 5), (7, 6)}
   (ii)



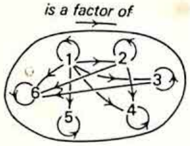
   (iii) → represents −1 (or *is one more than*).

(b) (i) {(0, 0), (1, 4), (2, 8), (3, 12)}
   (ii)



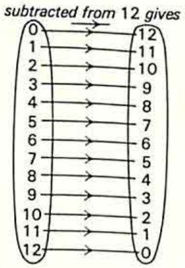
   (iii) → represents ×4 (or *is a quarter of*).

(c) (i) {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2,4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}
   (ii)



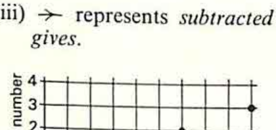
   (iii) → represents *is a factor of*.

(d) (i) {(0, 12), (1, 11), (2, 10), (3, 9), (4, 8), (5, 7), (6, 6), (7, 5), (8, 4), (9, 3), (10, 2), (11, 1), (12, 0)}
   (ii)



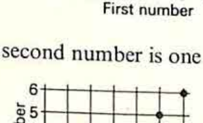
   (iii) → represents *subtracted from 12 gives.*

9. (a)



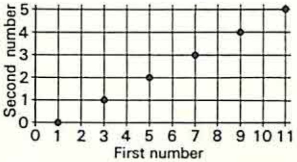
The second number is one third of the first.

(b)



The second number is a factor of the first.

(c)



The second number is a half of one less than the first.

10. (a) Domain: {0, 1, 2, 3, 4, 5, 6}
Range: {0, 1, 4, 9, 16, 25, 36}

(b) Domain: {Jide, Yusuf, Grace, Mary, Anieke}
Range: {5, 6, 8, 11}

(c) Domain: {0, 1, 2, 3}
Range: {0, 3, 6, 9}

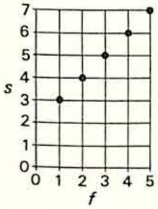
(d) Domain: {elephant, horse, dog}
Range: {horse, dog, chicken}

4. USING LETTERS TO SHOW RELATIONS

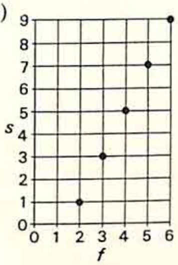
1. (a) $s = 3f$ (c) $s = f - 1$ (e) $s = 2f + 1$
(b) $s = f + 1$ (d) $s = \frac{1}{2}f$

2. (a) $s = \frac{1}{2}f$ (b) $s = 3f + 1$ (c) $s = f \times f$

3. (a)



(b)



4. (a) $1 \leqslant f \leqslant 9$ (c) $5 \leqslant f \leqslant 15$
(b) $0 \leqslant f \leqslant 6$ (d) $95 \leqslant f \leqslant 102$

5. (a) Function (b) Not a function

6. (a) Function (c) Not a function
(b) Not a function (d) Function

5. CONTINUOUS LINE GRAPHS

1. (a) 9 (c) 3
(b) 4 (d) 3

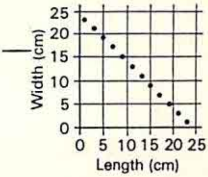
2. (a) 25 (b) 80

3. (a) $\frac{10}{90}$ (b) $\frac{40}{90}$ (c) $\frac{50}{90}$

9.

Length in cm	23	22	21	20	on to	2	1
Width in cm	1	2	3	4		22	23

Many other lengths and widths involving fractional values could be included.

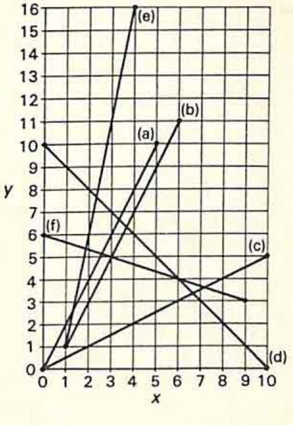


The points on the graph are not scattered at random but lie on a straight line.

11. It would be right to join the points by a curved line. For every time between 0 and 8 seconds there is a height.

12. It would be wrong to join the points. We cannot have fractions of a man.

13.

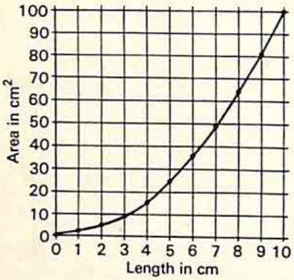


14. (a) $y = x + 2;\quad 0 \leqslant x \leqslant 5$
(b) $y = 4x;\quad 0 \leqslant x \leqslant 3$
(c) $y = \frac{1}{2}x + 1;\quad 0 \leqslant x \leqslant 12$
(d) $y = 7 - x;\quad 0 \leqslant x \leqslant 7$
(e) $y = 6 - \frac{1}{2}x;\quad 0 \leqslant x \leqslant 8$
(f) $y = 5;\quad 0 \leqslant x \leqslant 6$

ANSWERS

15.

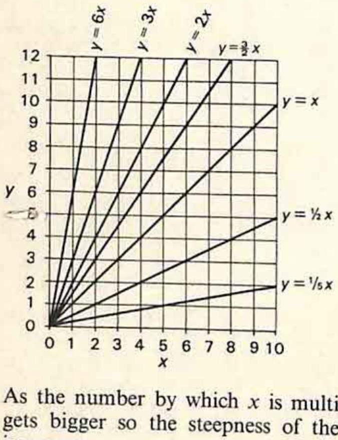
Length of edge (cm)	1	2	3	4	5	6	7	8	9	10
Area of square (cm^2)	1	4	9	16	25	36	49	64	81	100



It is reasonable to join the points with a smooth curve, as shown, because between the squares we have chosen there are many more. For example, between squares of edge length 4 cm and 5 cm we could have squares of edge length 4·2 cm, 4·38 cm, 4·813 cm, etc.
The equation of the line is $y = x \times x$ (or $y = x^2$).

16. See text below the exercise.

17.



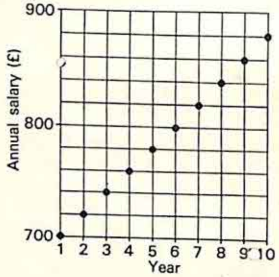
As the number by which x is multiplied gets bigger so the steepness of the line increases.

18. The equation of each line (reading from top to bottom) is: $y = x + 5$; $y = x + 2$; $y = x$; $y = x - 3$; $y = x - 5$.

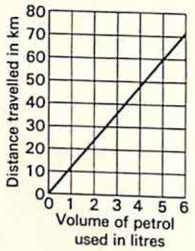
The graphs all have the same slope.
The lines above $y = x$ all have numbers added to x in their equations.
The lines below $y = x$ all have numbers subtracted from x in their equations.
The bigger the number added or subtracted, the further is the line from $y = x$.

19. The number by which x is multiplied is the same, e.g. $y = 3x + 1$ is parallel to $y = 3x - 2$.

20.



21.



22. (a) ~ 42 cm^2 (c) ~ 20 cm^2
(b) ~ 12 cm^2
(The symbol ~ represents 'approximately'.)

23. (a) ~ 7·1 cm (c) ~ 7·7 cm
(b) ~ 5·5 cm

24. The graph is a straight line.
$c \simeq 3d$ where c is the circumference, d the diameter, and $\simeq$ represents 'is approximately equal to'.

28. The circle is a special kind of ellipse. It is produced when the two pins are in the same position (i.e. only one pin is used).

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